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MINIMUM k-SELF-REPAIRING GRAPHS

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Minimum k-self-repairing graphs

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Abstract

We study in graphs properties related to fault-tolerance in case a node fails. A graph G is k-self-repairing, where k is a non-negative integer, if after the removal of any vertex no distance in the surviving graph increases by more than k. In the design of interconnection networks such graphs guarantee good fault-tolerance properties. In section 2, we give upper and lower bounds on the minimum number of edges of a k-self-repairing graph for prescribed k and n, where n is the order of the graph. In section 3, we prove that the problem of finding, in a k-self-repairing graph, a spanning k-self-repairing subgraph of minimum size is *NP-Hard*.

1 Introduction

Among the properties required in the design of efficient communication networks we consider those related to fault-tolerance. The least that must be guaranteed is that, after failure of some nodes or links, the surviving network still allows communication between all no-faulty nodes. This implies constraints on the connectivity of the corresponding graph. The k-connectivity (resp. k-edge-connectivity) is associated to the capability of a network to resist to the failure of any subset of $(k-1)$ nodes (resp. links).

Delay of communication in a network can be measured by the diameter of the underlying graph. It is important that surviving networks still allow communication with small delay. This implies constraints of bounded length for disjoint paths. The k-diameter of a k-connected graph and other variants of this parameter give a mean to study such properties (see [4]).

It is obvious that strengthening properties of fault-tolerance increases the number of links and therefore the cost of the network. One of the most major challenge is to design robust networks of efficient cost.

One approach consists in finding spanning subgraphs of a graph with the same good properties and fewer edges. When the property dealing about

is 2-connectivity, the problem is known as the 2-connected Steiner subgraph problem (see [2]).
In [1], the authors considered the minimum number of edges with diameter constraints.
A stronger property was studied by Farley and Proskurowski in [3]. The requirement is that no penalty on any distance after the removal of a node. That is no distance increases in the surviving graph. The authors determined the minimum number of edges in such graphs called self-repairing graphs and characterize the class of such minimum graphs in term of size.
In this paper, we consider graphs in which the removal of any node makes no distance in the safe graph increases by more that k.

Let G=(V,E) be a connected graph with m edges and n vertices. For a vertex x, $G-\{x\}$ denotes the subgraph of G induced by $V-\{x\}$. For x and y in V, we denote by $d_G(x,y)$ the length of a shortest path between x and y in G. We say that G is *k-self-repairing*, where k is a non-negative integer, if after the removal of any vertex the distances in the surviving graph increase by at most k. That is :

$$\forall x \in V, \forall\{y,z\} \subset V-\{x\}, d_{G-\{x\}}(y,z) \leq d_G(y,z)+k.$$

Note that such a graph is 2-connected and every pair of incident edges, called a transition, is on a cycle of length bounded by $k+4$.

Let $n \geq 4$ and $0 \leq k \leq n-4$. We define $g(n,k)$ as the minimum size of a k-self-repairing graph of order n. Biconnectivity implies that $g(n,k) \geq n$. For a fixed n, we have :

$$g(n,0) \geq g(n,1) \geq g(n,2) \geq \ldots \geq g(n,n-4).$$

It is easy to see that C_n, $n \geq 4$ is (n-4)-self-repairing of size n and then, $g(n,n-4)=n$.

Self-repairing graphs studied in [3] are those satisfying the definition above for $k=0$. The authors proved that $g(n,0)=2n-4$ and characterize such minimum graphs.

2 Bounds on the size of k-self-repairing graphs

Proposition 2.1 *: For $n \geq 6$ and $k \geq 1$,*

$$(n-1)+\frac{n-3}{k+1} \leq g(n,k) \leq (n+4)+2\lfloor\frac{n-2}{k+2}\rfloor.$$

Proof :

The lower bound must be proved only for k-self-repairing graphs having at least one vertex of degree 2 since otherwise the size of the graph is at least $\frac{3}{2}n$ which is better than the lower bound given in the proposition above.
Let x_0 be a vertex of degree 2. Consider a BFS tree T rooted at x_0. For an edge $[u, v]$ of G define $lmax([u, v])$ as $max\{L(u), L(v)\}$, where $L(u)$ is the *level* of vertex u in T, that is its distance from x_0. Let $e = [x, y]$ be an edge of T such that $lmax(e) \geq 2$. We assume that $L(x) > L(y)$. Let z be the parent of y. The transition defined by edges $[y, z], [y, x]$ is denoted by $tr(e)$. This transition is in a cycle of length at most $k + 4$. Indeed, after y, one can go down again through at most $\lfloor\frac{k+2}{2}\rfloor$ levels using edges of T before having to go back using an edge of $E - E(T)$. An edge of $E - E(T)$ used to make $tr(e)$ in a cycle of length at most $k + 4$ is called a *e-loopbackedge* (see figure 1).

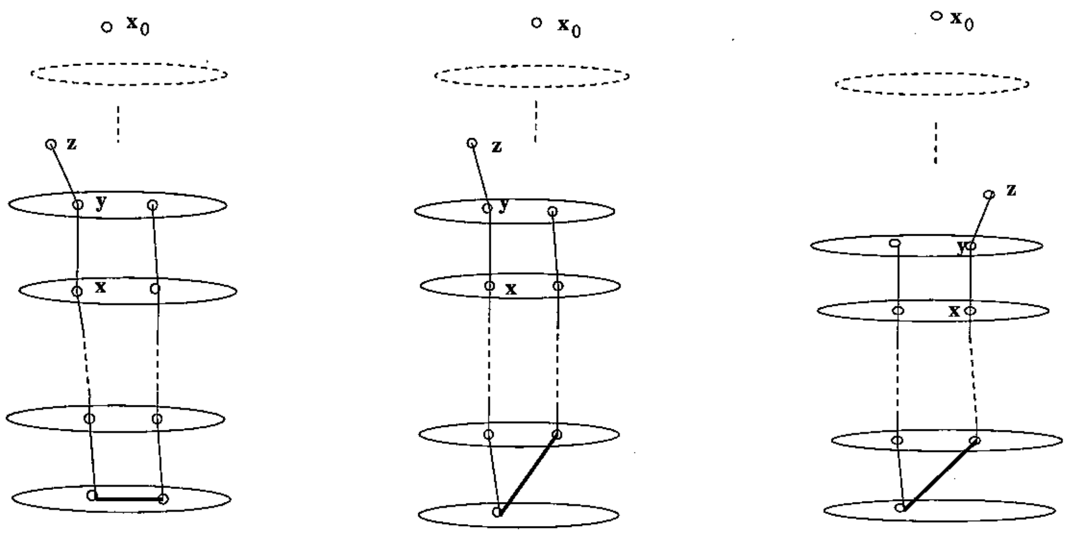

Figure 1: $e = [x, y]$ Edges shown in bold are *e-loopbackedges*

Each edge of $E-E(T)$ is an *e-loopbackedge* for at most $k+1$ edges e ($lmax(e) \geq 2$) of $E(T)$. Since $d_G(x_0) = 2$, the lower bound follows.

To prove the upper bound, we give constructions of family of such graphs.

First construction :

Two vertices u and v are said to be *related* if the following conditions are satisfied :

1. $d_G(u, v) = \lfloor\frac{k+4}{2}\rfloor$.

2. For every edge e incident to u, there exists a path of length at most $\lceil\frac{k+4}{2}\rceil$, containing e and connecting u and v.

3. For every edge e incident to v, there exists a path of length at most $\lceil\frac{k+4}{2}\rceil$, containing e and connecting u and v.

Let q and r be the two integers such that $n-(k+4)=q(\lfloor\frac{k+4}{2}\rfloor-1)+r$, where $r<\lfloor\frac{k+4}{2}\rfloor-1$. Start with a cycle G_0 of length $k+4$. Iterate q times the following step. Choose two related vertices in G_i. Connect them by a new path of $\lfloor\frac{k+4}{2}\rfloor-1$ new vertices. This gives a graph G_{i+1}. If r is non-zero, then, after the q iterations, choose again two related vertices u and v in G_q. Connect them by a new path of r new vertices.

The obtained graph is k-self-repairing of order n and has a size of at most $n+\frac{4n+k^2-2k-15}{2(k+1)}$ if n is odd and $n+\frac{4n+k^2-12}{2(k+2)}$ otherwise.

Second construction :

We give a second construction which is more complicated then the previous one but provides a better upper bound when k is odd and bounded by $O(\sqrt{n})$.

For $k\geq 1$, consider now the following family of k-self-repairing graphs of order n denoted $\mathcal{G}(n,k)$ constructed as follows (see figure 2) :

- Let t and r be the non-negative integers such that $\lfloor\frac{n}{2}\rfloor-1=t(k+2)+r$, with $0\leq r\leq(k+1)$.
- Set $l=t(k+2)$ and $\alpha=\lceil\frac{k+2}{2}\rceil$.
- Take two vertex-disjoint paths P_1 and P_2 each of length l. Set $P_1=[x_0,x_1,\ldots,x_l]$ and $P_2=[y_0,y_1,\ldots,y_l]$. Connect x_0 and y_0 by an edge.

Case k is at leat 2 :

- For all $i, 1\leq i\leq t$, connect $x_{i(k+2)}$ to $y_{i(k+2)}$ and $x_{i(k+2)-1}$ to $y_{i(k+2)-1}$.
- If k is even, connect $x_{i(k+2)+\alpha}$ to $y_{i(k+2)+\alpha}$ and $x_{i(k+2)+\alpha-1}$ to $y_{i(k+2)+\alpha-1}$, for all $i, 0\leq i\leq t-1$. If k is odd connect $x_{i(k+2)+\alpha}$ to $y_{i(k+2)+\alpha-1}$ and $x_{i(k+2)+\alpha-1}$ to $y_{i(k+2)+\alpha-2}$, for all $i, 0\leq i\leq t-1$.
- Set $n'=n-|V(P_1)\cup V(P_2)|$. We have $n'=n-2\lfloor\frac{n}{2}\rfloor+2r$ and then, $0\leq n'\leq 2k+2$ if n is even and, $1\leq n'\leq 2k+3$ otherwise. In the

following, we add n' new vertices and complete by necessary edges in order to obtain a k-self-repairing graph.

- If $n' \leq k+1$, place n' new vertices along a new path, say P, whose extremities are connected, one to x_l, the other to y_l . If $n' \leq k$, no more edge is needed and, the obtained k-self-repairing graph G verifies $|E(G)| = n + 4t$. If $n' = k+1$, add a chord in P as shown in figure 2 c)). This suffices to obtain a k-self-repairing graph G and we have, $|E(G)| = n + 1 + 4t$. If $n' = k+2$, place k vertices along a path as done for P, connect the two remaining vertices, say v_0 and w_0 by an edge and connect v_0 to x_0 and, w_0 to y_0 (figure 2 d)). No more edge is needed and the obtained k-self-repairing graph G verifies $|E(G)| = n + 2 + 4t$. If $k+3 \leq n' \leq 2k+2$, start with a construction as the one just previously obtained (figure 2 d)). This places $k+2$ vertices among the n'. Then, place the remaining vertices along a new path, P', whose extremities are connected, one to v_0, the other to w_0 (figure 2 e)). No more edge is needed and the obtained k-self-repairing graph G verifies $|E(G)| = n + 3 + 4t$. If $n' = 2k+3$, start with a construction as the one just previously obtained (figure 2 e)). This places $2k+2$ vertices among the n'. Then, place the remaining vertex on P or P' and add a new chord in the path where the remaining vertex has just been placed, as done in case c). We obtain a k-self-repairing graph G that verifies $|E(G)| = n + 4 + 4t$.

Case **k=1** (figure 3) :

- Connect x_1 to y_2, x_2 to y_1, x_l to y_l and, for all $i, 1 \leq i \leq (t-1)$, connect x_{3i} to y_{3i} and, x_{3i+1} to y_{3i-1} and y_{3i+2} and, x_{3i+2} to y_{3i+1}. If $n' = n - |V(P_1) \cup V(P_2)|$ is non-zero, place n' new vertices along a new path, connected as done above for P. If $n' = 1$ no more edge is needed. If $n' \geq 2$, let z_1 the extremity of P connected to x_l and $z_{n'}$ the one connected to y_l. Add the chord $[z_1, y_{l-1}]$. If $n' \leq 3$, no more edge is needed. If $n' = 4$, add the chord $[z_1, z_3]$ and then, no more edge is needed. If $n' = 5$, add the chord $[z_2, z_5]$ and then, no more edge is needed. The resulting graph is k-self-repairing and has a size of at most $n + 2 + 4t$.

A graph in $\mathcal{G}(n,k)$ is k-self-repairing and has a size of at most :

$$n + 4 + 4t = n + 4 + 4\lfloor \frac{\frac{n}{2} - 1}{k+2} \rfloor \leq (n+4) + 2\frac{n-2}{k+2}. \square$$

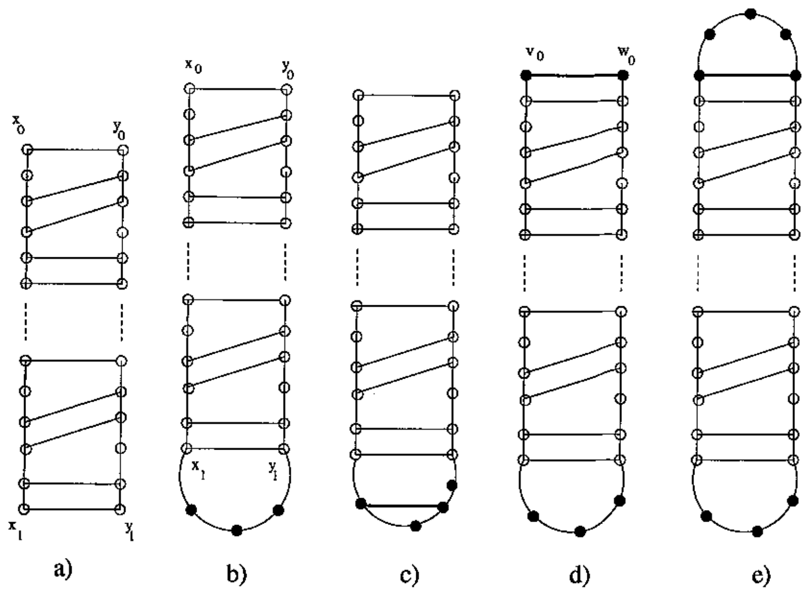

Figure 2: a) $n' = 0$. b) $1 \leq n' \leq k$. c) $n' = k+1$. d) $n' = k+2$. e) $k+3 \leq n' \leq 2k+2$.

3 Complexity of the problem of finding a minimum k-self-repairing spanning subgraph

In this section, we consider minimum k-self-repairing spanning subgraphs of a given graph. We denote $N_G(x)$ the open neighborhood of x in G, that is, the set of vertices of G adjacent to x in G.

First note that checking wether a graph is k-self-repairing can be done in polynomial time using the basic following algrithm :

Algorithm 3.1 *:*

Input : *A graph G, a positive integer k.*
Output : *Yes* if G is k-self-repairing , *No* if it is not.

```
begin
For each vertex x of G do
begin
    H ← G − x ;
    S ← N_G(x)
    For each pair of vertices {u, v} ⊆ S do
    begin
      compute d_H(u, v) ;
      if d_H(u, v) > k + 2, return No ;
    end;
```

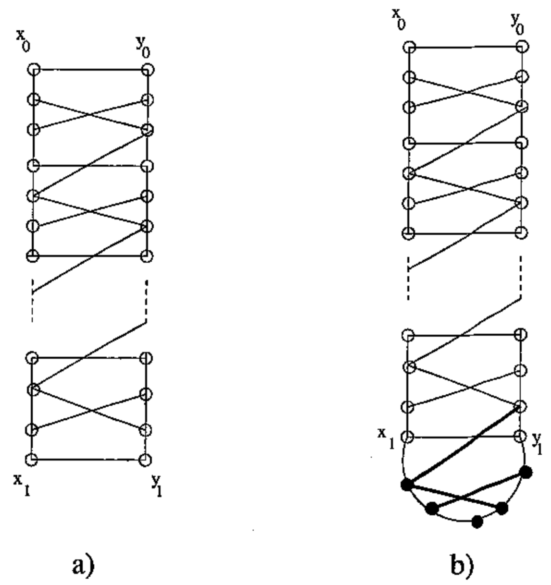

Figure 3: a) $n' = 0$. b) $n' = 5$.

end;
return *Yes* ;
end ;

Computing the distance between two vertices is done in polynomial time . For each vertex, we perform at most $d_G(x)(d_G(x) - 1)/2$ such computation. Therefore, the time of algorithm 3.1 is bounded by a polynomial of the size of G.

Problem 3.2 : Π.
Instance : *A positive integer $K \geq 2$, a K-self-repairing graph $\mathcal{G}$, a positive integer bound B.*
Question : *Does there exist a K-self-repairing spanning subgraph of $\mathcal{G}$ of size no more than B ?*

We shall prove that Π is NP-complete by proving a reduction from the following NP-complete *minimum spanning subgraph problem with diameter constraints* (MSSPDC) ([1]).

Problem 3.3 : MSSPDC.
Instance : *A positive integer $k \geq 2$, a graph G of diameter k, a positive integer bound b.*
Question : *Does there exist a spanning subgraph of G of diameter k and of size no more than b ?*

Theorem 3.4 *: Π is NP-complete.*

Proof

Π is in NP since, given a spanning subgraph of $\mathcal{G}$, one can verify, in polynomial time, wether it is K-self-repairing (using algorithm 3.1, for example) and then comparing its size to B.

Let $\mathcal{I}$ be an instance of MSSPDC that is positive integers k and b and a graph G of diameter k. Let $V = \{x_1, x_2, \ldots, x_n\}$ be the set of vertices of G. Consider the following instance $\mathcal{J}$ of Π :

1. The graph $\mathcal{G}$ of $\mathcal{J}$ is obtained by taking a copy of G and adding, from each vertex x_i a path P_i joining x_i and $t = k - 1$ new vertices. Let y_i be the second extremity of P_i. Add a new vertex z connected to all y_i (see figure 4).
2. $K = 3k - 4$ and $B = b + kn$, where n is the order of G.

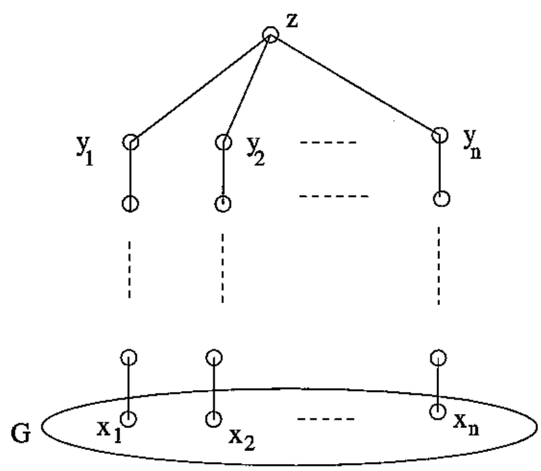

Figure 4:

The order of the size of $\mathcal{I}$ is at most $n^2 \log^2 n$. Since $k \leq n$ and $b \leq n^2$, the size of $\mathcal{J}$ is bounded by a polynomial in $n^2 \log^2 n$.

Now, we have to prove that $\mathcal{I}$ is a *yes-instance* of MSSPDC if and only if $\mathcal{J}$ is a *yes-instance* of Π.
Suppose there exists a spanning subgraph H of G of diameter k and of size bounded by b.
Consider the spanning subgraph $\mathcal{H}$ of $\mathcal{G}$ induced by all edges of H and all edges external to G. It is easy to see that $\mathcal{H}$ is K-self-repairing and of size at most B. Therefore, $\mathcal{J}$ is a *yes-instance* of Π.

Now, let $\mathcal{J}$ be an instance of Π constructed from an instance $\mathcal{I}$ of MSSPDC as described above. If $\mathcal{J}$ is a *yes-instance* of Π, there exists a spanning K-self-repairing graph $\mathcal{H}$ of $\mathcal{G}$ of size no more than B. The minimum degree in $\mathcal{H}$ is at least 2. This means that all edges external to G are in $\mathcal{H}$. Let H be the spanning subgraph of G induced on all edges of $E(\mathcal{H}) \cap E(G)$. Let $\{x_i, x_j\}$ be a pair of vertices of H. The transition of $\mathcal{H}$ defined by edges $[z, y_i]$, $[z, y_j]$ is in a cycle of length at most $K+4$. Every cycle containing y_i and y_j must include all vertices of paths P_i and P_j. Therefore, the distance in H between x_i and x_j is at most $K + 4 - 2k = k$ and then, H is a spanning subgraph of G of diameter k. The size of H, $|E(H)| = |E(\mathcal{H})| - kn \leq B - kn = b$ and, therefore $\mathcal{I}$ is a *yes-instance* of MSSPDC. □

The previous reduction shows that Π is NP-complete for $K = 3p+2$, $p \geq 0$. In fact, if we consider the same reduction from intances of MSSPDC, with $k \geq 3$, by setting $t = k-2$, $K = 3k-6$ and $B = b + n(k-1)$, we show that Π is NP-complete for $K = 3p, p \geq 1$. In the same way, considering instances of MSSPDC with $k \geq 4$, setting $t = k-3$, $K = 3k-8$ and $B = b + n(k-2)$, we show that Π is also NP-complete for $K = 3p+1, p \geq 1$. Therefore Π is NP-complete for all $K \geq 2$.

References

[1] L. Caccetta, I.B. Hartman, and J. Huang. Concerning the number of edges in a graph with diameter constraints. *Journal of Combinatorial Mathematics and Combinatorial Computing*, 36:161–173, 2001.

[2] C.R. Coullard, A. Rais, R.L. Rardin, and D.K. Wagner. Linear-time algorithms for the 2-connected steiner subgraph problem on special classes of graphs. *Networks*, 23:195–206, 1993.

[3] A.M. Farley and A. Proskurowski. Minimum self-repairing graphs. *Graphs Combin.*, 13:345–351, 1997.

[4] D.F. Hsu. On container width and length in graphs, groups and networks. *IEICE. Trans. Fundam E77*, 4:668–680, 1994.

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