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ORIENTATIONS OF LONG CYCLES IN BIPARTITE DIGRAPHS

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Orientations of long cycles in bipartite digraphs

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Abstract

Let D be a bipartite digraph with color classes X and Y such that $|X| = m \leq n = |Y|$. We give a sufficient size condition for D to contain an almost symmetric cycle of length $2m-2k$ ($0 \leq k < \frac{m}{2}$) and, in consequence, every orientation of a cycle of length $2m-2k$. The bound on size is best possible. We characterize all extremal digraphs for this problem. We thus obtain generalizations of the results by Wojda and Woźniak for hamiltonicity in bipartite digraphs.

keywords: digraph, bipartite digraph, cycle, cycle orientation, almost symmetric cycle

AMS Subject Classification: 05C20

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1 Terminology

With some exceptions specified below, we follow the standard terminology of [4] and [5].

Let G be a simple, undirected graph with the vertex set $V(G)$ and the edge set $E(G)$. Denote by $|G|$ the number of vertices in G, and by $||G||$ the number of edges in G. Let $G = (X, Y; E)$ denote a bipartite simple graph with the color classes X and Y and the set of edges E. Such a graph G is called *balanced* if X and Y have equal cardinalities.

For a digraph D, we denote by $V(D)$ the vertex set of D, by $A(D)$ the arc set of D, by $|D|$ the number of vertices in D, and by $||D||$ the number of arcs in D.

For $U \subset V(D)$, $D(U)$ denotes the subdigraph of D induced by U, while $D - U$ stands for the subdigraph of D induced by $V(G) \setminus U$. For subsets U and W of $V(D)$, by $e(U \to W)$ we denote the number of arcs from U to W, and $e(U, W) = e_D(U \to W) + e(W \to U)$. Let $d_D(x)$ denote the degree of the vertex x in D, i. e., $d_D(x) = e(x, V(D))$.

Let D_1 and D_2 be vertex disjoint digraphs. By $D_1 \rightleftharpoons D_2$ we denote the family of digraphs D such that $V(D) = V(D_1) \cup V(D_2)$ and for every $x, y \in V(D): \quad (x \to y) \in A(D)$ if and only if

(1) $x, y \in V(D_i)$ and $(x \to y) \in A(D_i)$ $(i = 1, 2)$ or

(2) $x \in V(D_i), y \in V(D_j), i \neq j$ and $(y \to x) \notin A(D)$ (the vertices of $V(D_1)$ are joined with the vertices of $V(D_2)$ by *antisymmetric arcs*).

Among the digraphs of the family $D_1 \rightleftharpoons D_2$ we distinguish one: $D_1 \to D_2$ with the arc set consisting of the arcs of D_1 and D_2, and all arcs from D_1 to D_2. Let us denote by $\mathcal{H}_{n,k}$ the family $K^*_{n-k-1} \rightleftharpoons K^*_{k+1}$, where K^*_m denotes a complete symmetric digraph with m vertices.

Finally, let $D_1 * D_2$ denote a digraph with $V(D_1 * D_2) = V(D_1) \cup V(D_2)$ and the arc set consisting of the arcs of D_1 and D_2, and all arcs between D_1 and D_2.

Let $D = (X, Y; A)$ denote a bipartite digraph with the color classes X and Y and the set of arcs A. Such a digraph D is called *balanced* if X and Y have equal cardinalities. Let $\overline{D}$ denote the complement of D to the complete bipartite symmetric digraph $K^*_{|X|,|Y|}$.

For a digraph $D = (X, Y; A)$, the *associated graph* $G(D)$ is defined to be an undirected bipartite graph with the same vertex set and the same color classes as D, and such that xy is an edge of $G(D)$ precisely when x and y are joined by a *symmetric arc* in D (i. e., $(x \to y) \in A(D)$ and $(y \to x) \in A(D)$). We shall tacitly use an obvious fact that $||G(D)|| \geq n^2 - s$ for a balanced bipartite digraph D with $2n$ vertices, whenever $||\overline{D}|| \leq s$.

Let $\mathcal{D}_{m,n,k} = \{\ D = (X, Y; A) :\ |X| = m \leq n = |Y|,\ X = U \cup W,$ $U = \{x \in X : d_D(x) = 2n\}$, $W = \{x \in X : d_D(x) = n\}$, $|W| = k + 1$, and each vertex of W is joined with all the vertices of Y by antisymmetric arcs $\}$. We distinguish special digraphs $D^1_{m,n,k}$ and $D^2_{m,n,k}$ of the family $\mathcal{D}_{m,n,k}$: in $D^1_{m,n,k}$ all antisymmetric arcs between the vertices of the sets W and Y are oriented from W to Y, while in $D^2_{m,n,k}$ they are oriented from Y to W.

A sequence $\epsilon = (\epsilon_1, ..., \epsilon_p)$, where $\epsilon_i \in \{-1, 1\}$, $1 \leq i \leq p$, is called the *orientation* of a cycle $C = x_1...x_px_1$ of D if $\epsilon_i = 1$ implies $(x_i \to x_{i+1}) \in A(D)$ and $\epsilon_i = -1$ implies $(x_{i+1} \to x_i) \in A(D)$ for every $i(\mathrm{mod} p)$. Then C is a *realization of* ϵ in D. Any realization in a digraph D of the orientation $\epsilon = (\epsilon_1, ..., \epsilon_p)$ is called a *strong cycle* of length p if $\epsilon_i\epsilon_{i+1} = 1$ for every $i(\mathrm{mod} p)$, and is denoted by $C^{\to}_p$. Let C^*_p denote a *symmetric cycle* of length p, that is a cycle with symmetric arcs only, and let $C^{*'}_p$ denote C^*_p minus one arc, which we call an *almost symmetric cycle* of length p (see Figure 1).

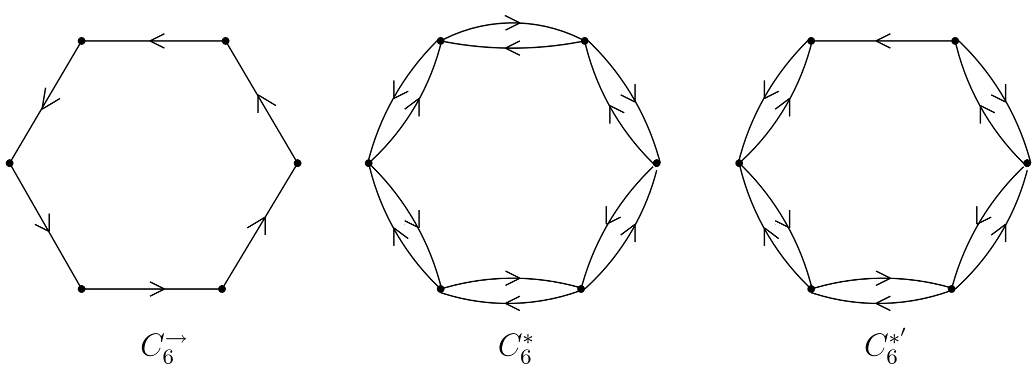

FIGURE 1

2 Results

The problem of finding the minimum number of arcs of a digraph D on n vertices, which guarantees that every orientation of a cycle of length $n-k$ (for $0 \le k < \frac{n}{2}$) is contained in D, was investigated in [3]. To solve this problem the authors proved the following theorem about symmetric and almost symmetric cycles in digraphs.

Theorem 1 ([3]) *Let D be a digraph on $n \geq 7$, $n \geq \frac{5}{2}k+6$ vertices and with at least $(n-k-1)(n-1)+k(k+1)$ arcs. Then:*

(1) D contains all symmetric cycles C_p^* for $3 \le p \le n-k-2$,

(2) D contains an almost symmetric cycle $C_{n-k-1}^{*'}$; moreover, if $n \geq 3k+6$ then D contains a symmetric cycle C_{n-k-1}^*,

(3) D contains an almost symmetric cycle $C_{n-k}^{*'}$ unless

(3a) D is one of the digraphs from $\mathcal{H}_{n,k}$,

(3b) $n = 3k+4$ and $D = T_{2k+3} * K_{k+1}^*$,

(3c) $n = 3k+2$ and $D = T_{2k+2} * K_k^*$,

where T_p is a tournament of order p.

As a consequence of the above theorem, the authors got the following corollary for all orientations of a cycle of length $n-k$.

Corollary 1.1 ([3]) *Let D be a digraph. Suppose that $|D| = n$, $n \geq 7$, $n \geq \frac{5}{2}k+6$ and $||D|| \geq (n-k-1)(n-1)+k(k+1)$. Then D contains every orientation of a cycle of length $n-k$ except the strong one in case $D = K_{n-k-1}^* \to K_{k+1}^*$ or $D = K_{k+1}^* \to K_{n-k-1}^*$.*

These results generalize the corresponding theorems by Heydemann, Sotteau and Thomassen [6], and Wojda [7], about hamiltonian cycles in digraphs.

Our purpose here is to prove analogous theorems in the case of bipartite digraphs.
Wojda and Woźniak investigated the existence of almost symmetric hamiltonian cycles in such digraphs.

Theorem 2 ([8]) *Let $D = (X, Y; A)$ be a bipartite digraph, where $|X| = |Y| = n$, and $||D|| \geq 2n^2 - 2n + 3$. Then D contains an almost symmetric hamiltonian cycle unless there is a vertex in D which is not incident to any symmetric arc of D.*

This result implies the following corollary about all orientations of hamiltonian cycles in balanced bipartite digraphs.

Corollary 2.1 ([8]) *Let $D = (X, Y; A)$ be a bipartite digraph, where $|X| = |Y| = n \geq 3$, and $||D|| \geq 2n^2 - n$. Then D contains every orientation of a hamiltonian cycle unless $D = D^1_{n,n,0}$ or $D = D^2_{n,n,0}$ (in both cases D does not contain a strong hamiltonian cycle).*

In the present paper we show generalizations of the above theorems to long cycles in general bipartite digraphs (not necessarily balanced). The conditions on size we obtain here are best possible. We characterize also all extremal digraphs for these problems.

The main result of this article is the following theorem.

Theorem 3 *Let $D = (X, Y; A)$ be a bipartite digraph with $|X| = m \leq n = |Y|$, where $m \geq \frac{1}{2}k^2 + \frac{5}{2}k + 6$. If*

$$||D|| \geq f(m, n, k) = 2mn - n(k + 1),$$

then:

(1) D contains all symmetric cycles C^*_{2p} for $2 \leq p \leq m - k - 1$,

(2) D contains an almost symmetric cycle $C^{*'}_{2m-2k}$ unless $||D|| = f(m, n, k)$ and $D \in \mathcal{D}_{m,n,k}$.

Theorem 3 asserts that either D contains an almost symmetric cycle $C^{*'}_{2m-2k}$, and hence every orientation of a cycle of length $2m - 2k$, or D is an exceptional digraph of the family $\mathcal{D}_{m,n,k}$. In the latter case, it is easy to see that every orientation of a cycle of length $2m - 2k$ is contained in D except the strong one in case $D = D^1_{m,n,k}$ or $D = D^2_{m,n,k}$. Hence we have the following corollary (observe that for $m = n \geq 6$ and $k = 0$ it is exactly Corollary 2.1).

Corollary 3.1 *Let $D = (X, Y; A)$ be a bipartite digraph with $|X| = m \leq n = |Y|$, where $m \geq \frac{1}{2}k^2 + \frac{5}{2}k + 6$. Suppose that $||D|| \geq f(m, n, k)$. Then D contains every orientation of a cycle of length $2m - 2k$ except the strong one in case $D = D^1_{m,n,k}$ or $D = D^2_{m,n,k}$.*

$\blacksquare$

3 Proof of the main theorem

The proof will be divided into two steps. We first show that Theorem 3 is true for $m = n$. In the second step we will consider situation when $m < n$.

Step 1. $m = n$

Part *(1)* of Theorem 3 is a simple corollary of the following theorem.

Theorem 4 ([1]) *Let $G = (X, Y; E)$ be a balanced bipartite undirected graph on $2n$ vertices, where $n \geq \frac{1}{2}l^2 + \frac{3}{2}l + 4$. If $||G|| \geq g(n, l) = n(n-l-1)+l+2$, then G contains cycles of all even lengths up to $2n - 2l$.*

Indeed, observe that for a balanced bipartite digraph D with $2n$ vertices and at least $f(n, n, k)$ arcs we have $||\overline{D}|| \leq n(k + 1)$. Hence, for the associated graph $G(D)$, $||G(D)|| \geq n^2 - n(k + 1) = n^2 - nk - n$.
For $n \geq \frac{1}{2}k^2 + \frac{5}{2}k + 6$, have $n \geq \frac{1}{2}(k + 1)^2 + \frac{3}{2}(k + 1) + 4$, and $n^2 - nk - n \geq n(n - k - 2) + k + 3 = g(n, k + 1)$, so can apply Theorem 4 to the graph $G(D)$ with $l = k+1$. Thus $C_{2p} \subset G(D)$ for all $2 \leq p \leq n - k - 1$, so that $C^*_{2p} \subset D$ for all $2 \leq p \leq n - k - 1$.

For the proof of part *(2)* of Theorem 3, we will proceed by induction on k.

Set $k = 0$. We want to show that if D is a balanced bipartite digraph with $|D| = 2n \geq 12$ and $||D|| \geq 2n^2 - n$ then there exists an almost symmetric hamiltonian cycle in D, unless $||D|| = 2n^2 - n$ and $D \in \mathcal{D}_{n,n,0}$.

Since $2n^2 - n \geq 2n^2 - 2n + 3$ for $n \geq 3$, then by Theorem 2, either D contains an almost symmetric cycle of length $2n - 2k$, or else there is a vertex in D which is not incident to any symmetric arc. In the latter case, the number of arcs of D implies that $D \in \mathcal{D}_{n,n,0}$.

Now assume the theorem holds for $k - 1$, $k \geq 1$; we will prove that it holds also for k.
Let us start with the observation that $f(n, n, k) - f(n - 1, n - 1, k - 1) = 3n - k - 2$. In the proof we shall consider two cases.

Case 1. There are two vertices $x \in X$ and $y \in Y$ in the digraph D such that $d_D(x) + d_D(y) \leq 3n - k - 2$.

Then $D - \{x, y\}$ is a balanced bipartite digraph with $2n-2$ vertices and at least $f(n,n,k) - (3n-k-2) = f(n-1,n-1,k-1)$ edges. By the inductive hypothesis there is an almost symmetric cycle $C^{*'}_{2(n-1)-2(k-1)} = C^{*'}_{2n-2k}$ contained in $D - \{x,y\}$, hence also in D, unless $D - \{x,y\} \in \mathcal{D}_{n-1,n-1,k-1}$. In the latter case, $d_D(x) + d_D(y) = 3n-k-2$ and x and y are not adjacent, because $||D - \{x,y\}|| = f(n-1,n-1,k-1)$. What's more, $X \cap V(D - \{x,y\}) = U \cup W$, where $U = \{v : d_{D-\{x,y\}}(v) = 2n-2\}$, $W = \{v : d_{D-\{x,y\}}(v) = n-1\}$, $|W| = k$, and each vertex of W is joined with all the vertices of $Y \setminus \{y\}$ by antisymmetric arcs.

It is easy to check that $C^{*'}_{2n-2k} \subset D$ if $d_D(x) \geq n$ (then at least one vertex of Y is joined with x by a symmetric arc). It thus remains to consider what happens when $d_D(x) \leq n-1$. Then $d_D(y) \geq 2n-k-1$, which implies that $e(y,U) \geq n-k$ and $e(y,W) \geq k+1$ if only $n > 2k$. It means that there are vertices: u in U and w in W, which are joined with the vertex y by symmetric arcs. In this case it is easy to find a cycle $C^{*'}_{2n-2k}$ contained in D passing through the vertices u, y, and w.

Case 2. For every pair of vertices $x \in X$ and $y \in Y$ in the digraph D, we have $d_D(x) + d_D(y) \geq 3n-k-1$.

By the already proved part *(1)* of Theorem 3, there is a symmetric cycle $C^*_{2n-2k-2} = x_1y_1x_2y_2\ldots x_{n-k-1}y_{n-k-1}x_1$ contained in D. We can partition the set of vertices of D into two subsets: vertices of the set $V(C^*_{2n-2k-2})$ and the remaining ones, which define a set denoted by V_{2k+2}.

Suppose that a cycle $C^{*'}_{2n-2k}$ is not contained in D. We need to consider several subcases, depending on the way the vertices of V_{2k+2} are connected.

Subcase 2.1. There are two vertices $x \in X \cap V_{2k+2}$ and $y \in Y \cap V_{2k+2}$ which are joined by a symmetric arc.

Since $C^{*'}_{2n-2k} \not\subset D$, we have $e(x,y_i) + e(y,x_i) \leq 2$, $i = 1,2,\ldots,n-k-1$, for otherwise we could extend the cycle $C^*_{2n-2k-2}$ to an almost symmetric cycle of length $2n-2k$ using connections between the vertices x_i, y, x, and y_i. It follows that

$e(x,V(C^*_{2n-2k-2})) + e(y,V(C^*_{2n-2k-2})) \leq 2(n-k-1)$. Hence $d_D(x)+d_D(y) \leq 2(n-k-1)+2(2k+2)$. But, by assumption made at the beginning of Case 2., $d_D(x)+d_D(y) \geq 3n-k-1$. Both inequalities imply $n \leq 3k+3$, contrary to the assumption $n \geq \frac{1}{2}k^2+\frac{5}{2}k+6$.

Subcase 2.2. No two vertices of the set V_{2k+2} are adjacent.
Let $x \in X \cap V_{2k+2}$ and $y \in Y \cap V_{2k+2}$. Similarly as in the previous case, since $C^{*'}_{2n-2k} \not\subset D$, we have $e(x,y_i)+e(y,x_i)+e(x,y_{i+1})+e(y,x_{i+1}) \leq 6$, $i=1,2,\ldots,n-k-1$ (reducing indices modulo $n-k-1$). If not, we could extend the cycle $C^*_{2n-2k-2}$ to an almost symmetric cycle of length $2n-2k$ using connections between the vertices x_i, y, x_{i+1}, y_i, x, and y_{i+1}. We deduce that $2d_D(x)+2d_D(y) \leq 6(n-k-1)$, contradicting $d_D(x)+d_D(y) \geq 3n-k-1$.

Subcase 2.3. There exist adjacent vertices in V_{2k+2}, but they are joined only by antisymmetric arcs.
Let $x \in X \cap V_{2k+2}$ and $y \in Y \cap V_{2k+2}$ be adjacent vertices. Again, since $C^{*'}_{2n-2k} \not\subset D$, we have $e(x,y_i)+e(y,x_i) \leq 3$, $i=1,2,\ldots,n-k-1$, for otherwise we could extend the cycle $C^*_{2n-2k-2}$ to an almost symmetric cycle of length $2n-2k$ using connections between the vertices x_i, y, x, and y_i. Hence $e(x,V(C^*_{2n-2k-2}))+e(y,V(C^*_{2n-2k-2})) \leq 3(n-k-1)$. It implies that $d_D(x)+d_D(y) \leq 3(n-k-1)+2(k+1)$, because x and y can be joined with other vertices of the set V_{2k+2} only by antisymmetric arcs. But $d_D(x)+d_D(y) \geq 3n-k-1$, so $d_D(x)+d_D(y) = 3n-k-1$ and the above inequality was, in fact, an equality, and we get that $d_{D(V_{2k+2})}(x) = d_{D(V_{2k+2})}(y) = k+1$. Applying the same argument to every pair of adjacent vertices of the set V_{2k+2}, we obtain that the induced digraph $D(V_{2k+2})$ is a balanced bipartite tournament $T_{k+1,k+1}$. What's more, for every $x \in X \cap V_{2k+2}$ and $y \in Y \cap V_{2k+2}$, $e(x,V(C^*_{2n-2k-2}))+e(y,V(C^*_{2n-2k-2})) = 3(n-k-1)$. Therefore $||D|| \leq 2(n-k-1)^2+3(n-k-1)(k+1)+(k+1)^2 = 2n^2-nk-n = f(n,n,k)$. It means that $||D|| = f(n,n,k)$ and vertices of $V(C^*_{2n-2k-2})$ induce a complete bipartite symmetric digraph $K^*_{n-k-1,n-k-1}$. There are exactly $3(n-k-1)(k+1)$ arcs between vertices of $T_{k+1,k+1}$ and $K^*_{n-k-1,n-k-1}$, and hence some of these arcs are symmetric. Without loss of generality we can assume that there is a vertex $v \in Y \cap V(T_{k+1,k+1})$ joined

by a symmetric arc with $x_i \in X \cap V(K^*_{n-k-1,n-k-1})$ for some $i \in \{1, \ldots, n-k-1\}$. Since $C^{*'}_{2n-2k} \not\subset D$, there are no symmetric arcs between vertices of the sets $X \cap V(T_{k+1,k+1})$ and $Y \cap V(K^*_{n-k-1,n-k-1})$. Thus

$$e(X \cap V(T_{k+1,k+1}), Y \cap V(K^*_{n-k-1,n-k-1})) \leq (k+1)(n-k-1).$$

Of course,

$$e(Y \cap V(T_{k+1,k+1}), X \cap V(K^*_{n-k-1,n-k-1})) \leq 2(k+1)(n-k-1).$$

The number of arcs in D implies that the above inequalities are, in fact, equalities, and $D \in \mathcal{D}_{n,n,k}$.

Step 2. $m < n$

First order the vertices of $Y = \{y^1, \ldots, y^n\}$ so that $d_D(y^1) \leq \ldots \leq d_D(y^n)$. Put $Y' = \{y^1, \ldots, y^{n-m}\}$.

We shall consider two cases depending on the degree of the vertex y^{n-m}.

Case 1. $d_D(y^{n-m}) \geq 2m - k$

Then $D - Y'$ is a balanced bipartite digraph on $2m$ vertices such that $||D - Y'|| \geq m(2m-k) > f(m,m,k)$ since $m > 0$. From what has already been proved in Step 1., we conclude that $D - Y' \supset C^*_{2p}$ for all $2 \leq p \leq m-k-1$ and $D - Y' \supset C^{*'}_{2m-2k}$, which completes the proof in this case.

Case 2. $d_D(y^{n-m}) \leq 2m - k - 1$

Now we have $||D - Y'|| \geq f(m,n,k) - (n-m)(2m-k-1) = 2m^2 - mk - m = f(m,m,k)$, which again implies $D - Y' \supset C^*_{2p}$ for all $2 \leq p \leq m-k-1$, and also $D - Y' \supset C^{*'}_{2m-2k}$ unless $||D - Y'|| = f(m,m,k)$ and $D - Y' \in \mathcal{D}_{m,m,k}$. Therefore it remains to consider the situation when $X = U \cup W$, where $U = \{v : d_{D-Y'}(v) = 2m\}$, $W = \{v : d_{D-Y'}(v) = m\}$, $|W| = k+1$, and each vertex of W is joined with all the vertices of $Y \setminus Y'$ by antisymmetric arcs. Then every vertex of Y' has degree in D equal $2m-k-1$ since $||D - Y'|| = f(m,m,k)$.

Suppose now that any almost symmetric cycle of length $2m-2k$ is not contained in D. Consequently, no vertex of Y' can be joined with both a vertex of U and a vertex of W by symmetric arcs, for otherwise we could simply construct a cycle $C^{*\prime}_{2m-2k}$ in D. Hence the assumption: $d_D(y^i) = 2m - k - 1$ for all $1 \leq i \leq n - m$, implies (for $m > 2k + 2$) that every vertex of Y' is joined with all vertices of U by symmetric arcs and with all vertices of W by antisymmetric arcs. It follows that $D \in \mathcal{D}_{m,n,k}$, which completes the proof.

■

4 Conclusion

We expect that the assumption $m \geq \frac{1}{2}k^2 + \frac{5}{2}k + 6$ in Theorem 3 may be relaxed; that is, that m can be assumed to be linear compared to k. The present quadratic bound on m is simply a consequence of the assumptions of Theorem 4, which we use in the proof of part *(1)* of Theorem 3.
The essential assumption on m in the proof of part *(2)* of Theorem 3 is $m > 3k + 3$ (see Subcase 2.1. in Step 1.). Thus the condition on m would be weakened automatically if only we could improve Theorem 4, for example by proving the following conjecture which was stated in [2].

Conjecture 1 *Let $G = (X, Y; E)$ be a balanced bipartite undirected graph on $2n$ vertices, where $n \geq 2k + 2$. If $||G|| \geq n(n - k - 1) + k + 2$, then G contains cycles of all even lengths up to $2n - 2k$.*

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