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Cycle Ingénieur – 2ème année

Département Informatique

Verification and Validation

Part IV : An Introduction to Testing

Burkhart Wolff

Département Informatique

Université Paris-Saclay / LMF

Recall: Validation and Verification

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  Do we build the system right ?   Is it « correct » ?

How to do Validation ?

- Tests and Experiments over Systems (Integrated artefacts consisting of software and hardware …)

How to do Verification ?

- Test and Proof on the basis of formal specifications (e.g., à la OCL, MOAL, ACSL, ... !) against programs or systems ...

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  - *on the other side – you can't test infinitely, and verification is again 10 times more costly than thoroughly testing !*

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    - a lot of knowledge over methods, tools, and tool chains ...

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  - *ideally*: testing is complete if a certain criteria, the adequacy criteria is reached.

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  - non-deterministic programs with no control over the non-determinism.

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  - experiment with the real behaviour
  - essentially used for post-hoc ananalysis and debugging

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- **reactive sequence**: testing components by sequences of steps, but these sequences represent communication where later parts in the seqience depend on what has been earlier cummunicated

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  control-flow, data-flow paths or by using symbolic executions.
- **both**: (also: grey-box testing).

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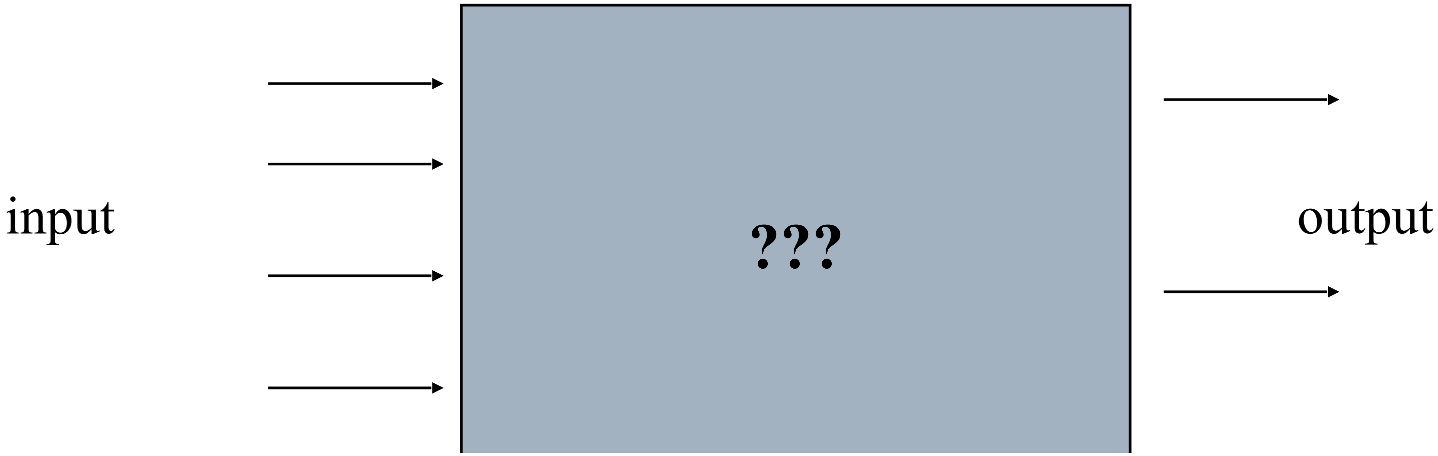

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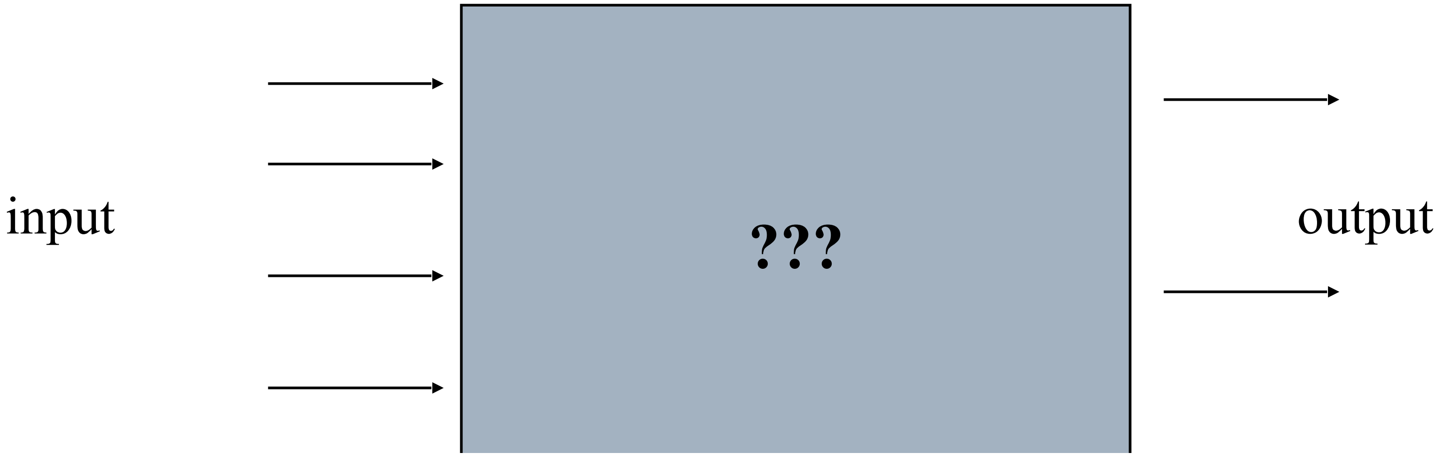

we focus on what the program *should* do !!!

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Give a specification, and develop a test set ...

Functional Unit Test : An Example

The specification in UML/MOAL:

```
Triangles
----------------------------------------
 a, b, c: Integer
----------------------------------------
- mk(Integer,Integer,Integer):Triangle
- is_Triangle(): {equ (*equilateral*),
                  iso (*isosceles*),
                  arb (*arbitrary*)}
```

Functional Unit Test : An Example

We add the constraints of the analysis:

inv $0<a \land 0<b \land 0<c$
inv $c \le a+b \land a \le b+c \land b \le c+a$

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operation t.is_Triangle():

post $t.a=t.b \land t.b=t.c \longrightarrow result=equ$
post $(t.a \neq t.b \lor t.b \neq t.c) \land (t.a=t.b \lor t.b=t.c \lor t.a=t.c)) \longrightarrow result=iso$
post $(t.a \neq t.b \lor t.b \neq t.c \lor t.a \neq t.c)) \longrightarrow result=arb$

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check_C(); check_C_1(); ... ; check_C_n();

assert(P(self,a_1,...,a_n));

result=run_m(self,a_1,...,a_n);

assert(Q(self,a_1,...,a_n,result));

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- ... work on real data and are extremely helpful for post-hoc crash-analysis, debugging, and forensics.
- Runtime-tests conflict with efficiency
- But: they are NOT particularly useful during development, where we need systematic test-data EARLY.

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  - ... gives clear criteria on the question: „did we test enough" ?

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  Note that we define mk(0,0,0) to invalid, as well as all other invalid triangles …

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- impossible triangles and their permutations :

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- Would we have to consider negative values?

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- Ouf, is there a systematic and automatic way to compute all these tests ?
- Can we avoid hand-written test-scripts ? Avoid the task to maintain them ?
- And the question remains:

When did we test „enough" ?

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(* see semantics of MOAL in Part III *)

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- From $mk(x,y,z) \neq null$ (see $pre_{isTriangle}$) and from $inv_{Triangle}(\sigma)$ and $mk(x,y,z) \in Triangle\ (\sigma)$ follows that:

  $0<x \wedge 0<y \wedge 0<z \ \wedge\ x \leq y+z \ \wedge\ y \leq x+z \ \wedge\ z \leq x+y \qquad (\equiv inv)$

Revision: Boolean Logic + Some Basic Rules

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- $\neg(a \wedge b) = \neg a \vee \neg b$ (* deMorgan1 *)
- $\neg(a \vee b) = \neg a \wedge \neg b$ (* deMorgan2 *)
- $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
- $\neg(\neg a) = a$, $a \vee \neg a = T$, , $a \wedge \neg a = F$,
- $a \wedge b = b \wedge a$; $a \vee b = b \vee a$
- $a \wedge (b \wedge c) = (a \wedge b) \wedge c$
- $a \vee (b \vee c) = (a \vee b) \vee c$
- $a \longrightarrow b = (\neg a) \vee b$
- $(a = b \wedge P(a)) = P(b)$ (* one point rule *)

- let x = E in C(x) = C(E) (* let elimination *)
- if c then C else D = $(c \wedge C) \vee (\neg c \wedge D) = (c \longrightarrow C) \wedge (\neg c \longrightarrow D)$

Test-Data Generation

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(* see semantics d'un appel de methopde, in MOAL II, page 22. *)

Some Facts:

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Some Facts:

- $\mathtt{arb} \neq \mathtt{equ} \neq \mathtt{iso}$
- $post_{isTriangle}(mk(x,y,z),r)(\sigma,\sigma)$ can be simplified to:

  $(x=y \wedge y=z \longrightarrow r=equ) \wedge$

  $((x\neq y \vee y\neq z) \wedge (x=y \vee y=z \vee x=z) \longrightarrow r=iso) \wedge$

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$\Longrightarrow$ (* the discussed facts *)

Test-Data Generation

- Summing up:

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$\equiv$ $inv_{Triangle}(\sigma) \wedge pre_{isTriangle}(mk(x,y,z))(\sigma) \wedge$
$inv_{Triangle}(\sigma') \wedge post_{isTriangle}(mk(x,y,z),r)(\sigma,\sigma')$

$\Longrightarrow$ (* the discussed facts *)

$$
\begin{array}{l}
\mathtt{inv} \wedge \\
(x = y \wedge y = z \longrightarrow r = equ) \wedge \\
((x \neq y \vee y \neq z) \wedge (x = y \vee y = z \vee x = z) \longrightarrow r = iso) \wedge \\
(x \neq y \wedge y \neq z \wedge x \neq z \longrightarrow r = arb)
\end{array}
$$

Test-Data Generation

- Recall the test specification:

$$\texttt{inv} \wedge (x=y \wedge y=z \longrightarrow r=equ) \wedge$$
$$((x\neq y \vee y\neq z) \wedge (x=y \vee y=z \vee x=z) \longrightarrow r=iso) \wedge$$
$$(x\neq y \wedge y\neq z \wedge x\neq z \longrightarrow r=arb)$$

$\equiv$ (* elimination $\longrightarrow$, deMorgan*)

$$\texttt{inv} \wedge$$
$$(x\neq y \vee y\neq z \vee r=equ) \wedge$$
$$((x=y \wedge y=z) \vee (x\neq y \wedge y\neq z \wedge x\neq z) \vee r=iso) \wedge$$
$$(x=y \vee y=z \vee x=z \vee r=arb)$$

Test-Data Generation

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- This first part of the calculation could be called

  PURIFICATION

  We eliminate UML, object-orientation, MOAL etcpp and reduce it to the pure logical core ...

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Now, under which precise conditions do we have

  - r = iso
  - r = arb
  - r = equ ???

Test-Data Generation

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  PURIFICATION

  We eliminate UML, object-orientation, MOAL etcpp and reduce it to the pure logical core …

  Can we transform the spec into the form
  - $A_1 \land \ldots \land A_i \land r = iso$
  - $B_1 \land \ldots \land B_k \land r = arb$
  - $C_1 \land \ldots \land C_l \land r = equ$???

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  PURIFICATION

  We eliminate UML, object-orientation, MOAL etcpp and reduce it to the pure logical core …

  Can we transform the spec into a

  Disjunctive Normal Form (DNF) ?

Excursion

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- Generalized Distribution Laws:

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$$(A_1 \vee A_2) \wedge (B_1 \vee B_2) \quad = (A_1 \wedge (B_1 \vee B_2)) \vee (A_2 \wedge (B_1 \vee B_2))$$

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$$(A_1 \vee A_2 \vee A_3) \wedge (B_1 \vee B_2 \vee B_3) \wedge (C_1 \vee C_2 \vee C_3)$$

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$$= \ldots$$

Excursion

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$(A_1 \vee A_2) \wedge (B_1 \vee B_2) \quad = (A_1 \wedge (B_1 \vee B_2)) \vee (A_2 \wedge (B_1 \vee B_2))$

$= (A_1 \wedge B_1) \vee (A_2 \wedge B_1) \vee (A_1 \wedge B_2) \vee (A_2 \wedge B_2)$

$(A_1 \vee A_2 \vee A_3) \wedge (B_1 \vee B_2 \vee B_3) \wedge (C_1 \vee C_2 \vee C_3)$

$= \ldots$

$= (A_1 \wedge B_1 \wedge C_1) \vee (A_1 \wedge B_1 \wedge C_2) \vee (A_1 \wedge B_1 \wedge C_3) \vee$

$(A_2 \wedge B_1 \wedge C_1) \vee (A_2 \wedge B_1 \wedge C_2) \vee (A_2 \wedge B_1 \wedge C_3) \vee$

...

$(A_1 \wedge B_3 \wedge C_3) \vee (A_2 \wedge B_3 \wedge C_3) \vee (A_3 \wedge B_3 \wedge C_3)$

Test-Data Generation

Test-Data Generation

- Recall the test specification:

  ...

  $\equiv$ inv $\wedge$

  $(x \neq y \vee y \neq z \vee r = equ) \wedge$

  $(x = y \vee y = z \vee x = z \vee r = arb) \wedge$

  $((x = y \wedge y = z) \vee (x \neq y \wedge y \neq z \wedge x \neq z) \vee r = iso)$

  $\equiv$

Test-Data Generation

- Recall the test specification:

  ...

  $\equiv$ inv ∧

  $\boxed{\begin{array}{l}(x \neq y \lor y \neq z \lor r = equ) \land \\ (x = y \lor y = z \lor x = z \lor r = arb) \land\end{array}}$

  $((x = y \land y = z) \lor (x \neq y \land y \neq z \land x \neq z) \lor r = iso)$

  $\equiv$

distrib

Test-Data Generation

- Recall the test specification:

  ...

$$\equiv \; inv \land$$
$$\boxed{(x \neq y \lor y \neq z \lor r = equ) \land (x = y \lor y = z \lor x = z \lor r = arb) \land}$$
$$((x = y \land y = z) \lor (x \neq y \land y \neq z \land x \neq z) \lor r = iso)$$

distrib

$$\equiv$$

$$inv \land$$
$$((x \neq y \land x = y) \lor (x \neq y \land y = z) \lor (x \neq y \land x = z) \lor (x \neq y \land r = arb)) \lor$$
$$((y \neq z \land x = y) \lor (y \neq z \land y = z) \lor (y \neq z \land x = z) \lor (y \neq z \land r = arb)) \lor$$
$$((r = equ \land x = y) \lor (r = equ \land y = z) \lor (r = equ \land x = z) \lor (r = equ \land r = arb)) \lor$$
$$((x = y \land y = z) \lor (x \neq y \land y \neq z \land x \neq z) \lor r = iso)$$

Test-Data Generation

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- Recall the test specification:

  ...

Test-Data Generation

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  ...

  $\equiv$ `inv` $\wedge$

  $(x \neq y \vee y \neq z \vee r = equ) \wedge$

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  $((x = y \wedge y = z) \vee (x \neq y \wedge y \neq z \wedge x \neq z) \vee r = iso)$

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  ...

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  $((x = y \wedge y = z) \vee (x \neq y \wedge y \neq z \wedge x \neq z) \vee r = iso)$

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  ...

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  $(x \neq y \vee y \neq z \vee r = equ) \wedge$

  $(x = y \vee y = z \vee x = z \vee r = arb) \wedge$

  $((x = y \wedge y = z) \vee (x \neq y \wedge y \neq z \wedge x \neq z) \vee r = iso)$

  $\equiv$ (* elimination contradictions *)

Test-Data Generation

- Recall the test specification:

  ...

  $\equiv$ inv $\wedge$
  $(x \neq y \vee y \neq z \vee r = equ) \wedge$
  $(x = y \vee y = z \vee x = z \vee r = arb) \wedge$
  $((x = y \wedge y = z) \vee (x \neq y \wedge y \neq z \wedge x \neq z) \vee r = iso)$

  $\equiv$ (* elimination contradictions *)
  inv $\wedge$
  $((x \neq y \wedge x = y) \vee (x \neq y \wedge y = z) \vee (x \neq y \wedge x = z) \vee (x \neq y \wedge r = arb) \vee$
  $(y \neq z \wedge x = y) \vee (y \neq z \wedge y = z) \vee (y \neq z \wedge x = z) \vee (y \neq z \wedge r = arb) \vee$
  $(r = equ \wedge x = y) \vee (r = equ \wedge y = z) \vee (r = equ \wedge x = z) \vee (r = equ \wedge r = arb)) \vee$
  $((x = y \wedge y = z) \vee (x \neq y \wedge y \neq z \wedge x \neq z) \vee r = iso)$

Test-Data Generation

- Recall the test specification:

  ...

  ≡ (* elimination contradictions *)

$$\begin{aligned}&\texttt{inv} \wedge\\ &\quad\big((x\neq y \wedge y=z)\vee(x\neq y \wedge x=z)\vee(x\neq y \wedge r=arb) \vee\\ &\quad\ (y\neq z \wedge x=y)\vee(y\neq z \wedge x=z)\vee(y\neq z \wedge r=arb) \vee\\ &\quad\ (r=equ\wedge x=y)\vee(r=equ\wedge y=z)\vee(r=equ\wedge x=z)\big) \wedge\\ &\quad\big((x=y \wedge y=z) \vee (x\neq y \wedge y\neq z \wedge x\neq z) \vee r=iso\big)\end{aligned}$$

Test-Data Generation

- $\equiv$ (* generalized distribution 2nd/3rd ((9 * 3 = 27 cases !)*)

$$
\begin{array}{l}
\texttt{inv} \land \\
\quad ((x\neq y \land y=z \land x=y \land y=z) \lor (x\neq y \land x=z \land x=y \land y=z) \lor (x\neq y \land r=arb \land x=y \land y=z) \lor \\
\quad (y\neq z \land x=y \land x=y \land y=z) \lor (y\neq z \land x=z \land x=y \land y=z) \lor (y\neq z \land r=arb \land x=y \land y=z) \lor \\
\quad (r=equ \land x=y \land x=y \land y=z) \lor (r=equ \land y=z \land x=y \land y=z) \lor (r=equ \land x=z \land x=y \land y=z)) \lor \\
\quad ((x\neq y \land y=z \land x\neq y \land y\neq z \land x\neq z) \lor (x\neq y \land x=z \land x\neq y \land y\neq z \land x\neq z) \lor (x\neq y \land r=arb \\
\quad \land x\neq y \land y\neq z \land x\neq z) \lor (y\neq z \land x=y \land x\neq y \land y\neq z \land x\neq z) \lor (y\neq z \land x=z \land x\neq y \land y\neq z \land \\
\quad x\neq z) \lor (y\neq z \land r=arb \land x\neq y \land y\neq z \land x\neq z) \lor (r=equ \land x=y \land x\neq y \land y\neq z \land x\neq z) \lor (\\
\quad r=equ \land y=z \land x\neq y \land y\neq z \land x\neq z) \lor (r=equ \land x=z \land x\neq y \land y\neq z \land x\neq z)) \lor \\
\quad ((x\neq y \land y=z \land r=iso) \lor (x\neq y \land x=z \land r=iso) \lor (x\neq y \land r=arb \land r=iso) \\
\quad \lor (y\neq z \land x=y \land r=iso) \lor (y\neq z \land x=z \land r=iso) \lor (y\neq z \land r=arb \land r=iso) \lor \\
\quad (r=equ \land x=y \land r=iso) \lor (r=equ \land y=z \land r=iso) \lor (r=equ \land x=z \land r=iso))
\end{array}
$$

Test-Data Generation

- $\equiv$ (* elimination of the contradictions and redundancies *)

$$
\begin{array}{l}
\mathtt{inv}\ \wedge \\
\big((x\neq y \wedge y=z \wedge x=y \wedge y=z) \vee (x\neq y \wedge x=z \wedge \\
\qquad x=y \wedge y=z) \vee (x\neq y \wedge r=arb \wedge x=y \wedge y=z) \ \vee \\
\ (y\neq z \wedge x=y \wedge x=y \wedge y=z) \vee (y\neq z \wedge x=z \wedge \\
\qquad x=y \wedge y=z) \vee (y\neq z \wedge r=arb \wedge x=y \wedge y=z) \ \vee \\
\ (r=equ \wedge x=y \wedge x=y \wedge y=z) \vee \underline{(r=equ \wedge} \\
\qquad \underline{y=z \wedge x=y \wedge y=z)} \vee \underline{(r=equ \wedge x=z \wedge x=y \wedge y=z)}\big) \vee \\
\big((x\neq y \wedge y=z \wedge x\neq y \wedge y\neq z \wedge x\neq z) \vee (x\neq y \wedge x=z \wedge x\neq y \wedge y\neq z \wedge x\neq z) \vee (x\neq y \wedge r=arb \\
\wedge\ \underline{x\neq y \wedge} y\neq z \wedge x\neq z) \vee (y\neq z \wedge x=y \wedge x\neq y \wedge y\neq z \wedge x\neq z) \vee (y\neq z \wedge x=z \wedge x\neq y \wedge y\neq z \wedge \\
x\neq z) \vee \underline{(y\neq z \wedge r=arb \wedge x\neq y \wedge y\neq z \wedge x\neq z)} \vee (r=equ \wedge x=y \wedge x\neq y \wedge y\neq z \wedge x\neq z) \vee (\\
r=equ \wedge y=z \wedge x\neq y \wedge y\neq z \wedge x\neq z) \vee (r=equ \wedge x=z \wedge x\neq y \wedge y\neq z \wedge\ x\neq z)\big) \vee \\
\big((x\neq y\ \wedge\ y=z \wedge r=iso) \vee (x\neq y\ \wedge\ x=z \wedge r=iso) \vee (x\neq y \wedge r=arb \wedge r=iso) \\
\ \vee (y\neq z \wedge x=y \wedge r=iso) \vee (y\neq z \wedge x=z \wedge r=iso) \vee (y\neq z \wedge r=arb \wedge r=iso) \ \vee \\
\ (r=equ \wedge x=y \wedge r=iso) \vee (r=equ \wedge y=z \wedge r=iso) \vee (r=equ \wedge x=z \wedge r=iso)\big)
\end{array}
$$

Test-Data Generation

Test-Data Generation

- ≡ (* cleanup, distribution *)

```
(inv ∧ x=y ∧ x=y ∧ y=z ∧ r=equ) ∨              (1)
(inv ∧ x≠y ∧ y≠z ∧ x≠z ∧ r=arb ) ∨             (2)
(inv ∧ x≠y ∧ y=z ∧ r=iso) ∨                    (3)
(inv ∧ x≠y ∧ x=z ∧ r=iso) ∨                    (4)
(inv ∧ y≠z ∧ x=y ∧ r=iso) ∨                    (5)
(inv ∧ y≠z ∧ x=z ∧ r=iso)                      (6)
```

Test-Data Generation

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```
(inv ∧ x=y ∧ x=y ∧ y=z ∧ r=equ) ∨          (1)
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(inv ∧ x≠y ∧ x=z ∧ r=iso) ∨                (4)
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```

- Test-Case-Construction by DNF Method

Test-Data Generation

- ≡ (* cleanup, distribution *)

```
(inv ∧ x=y ∧ x=y ∧ y=z ∧ r=equ) ∨           (1)
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(inv ∧ x≠y ∧ x=z ∧ r=iso) ∨                 (4)
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```

- Test-Case-Construction by DNF Method

  yields six abstract test cases

  relating input x y z to output r

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(inv ∧ x=y ∧ x=y ∧ y=z ∧ r=equ) ∨          (1)
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```

- Test-Case-Construction by DNF Method

  yields six abstract test cases

  relating input x y z to output r

- Note: In general, output r is not necessarily uniquely defined as in our example ...
  The spec can be non-deterministic admitting several results.

Test-Data Generation

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- Test-Data-Selection:
  For each abstract test-case, we construct one concrete test, by choosing values that make the abstract test case true (« that satisfies the abstract test case »)

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case	x	y	z	result
(1)	3	3	3	equ
(2)	3	4	6	arb
(3)	4	5	5	iso
(4)	5	4	5	iso
(5)	5	5	4	iso
(6)	4	3	4	iso

Test-Data Generation

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  - Any basic predicate ("literal") has been used at least one time
    - … provided it is not contradictory ("A=False")
    - … provided that it is not redundant ("A=True")
    - … provided it is not implied by another literal, i.e. it is subsumed ("B $\longrightarrow$ A")

Test-Data Generation

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- A First Summary on the Test-Generation Method:

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    one concrete test-input is constructed.
    (**ISO/IEC/IEEE** 29119 calls this: Equivalence class testing)
- Remark: During Codiung phase, when the Spec does not change, the test-data-selection can be repeated easily creating always different test sets ...

Test-Data Generation

- Variants:
  - Alternative to PHASE II (DNF construction): Predicate Abstraction and Tableaux-Exploration.

    Reconsider the (purified) specification:

$$
\begin{array}{l}
\mathtt{inv} \land \\
(x=y \land y=z \longrightarrow r=equ) \land \\
((x \neq y \lor y \neq z) \land (x=y \lor y=z \lor x=z) \longrightarrow r=iso) \land \\
(x \neq y \land y \neq z \land x \neq z \longrightarrow r=arb)
\end{array}
$$

    It is possible to abstract this spec to a fairly small number of „base predicates" ... They should be logically independent and not contain the output variable...

Test-Data Generation

- Variants:
  - Alternative to PHASE II (DNF construction): Predicate Abstraction and Tableaux-Exploration.

    Reconsider the (purified) specification:

$$
\begin{array}{l}
\texttt{inv} \wedge \\
(\texttt{A} \wedge \texttt{B} \longrightarrow \texttt{r=equ}) \wedge \\
((\neg \texttt{A} \vee \neg \texttt{B}) \wedge (\texttt{A} \vee \texttt{B} \vee \texttt{C}) \longrightarrow \texttt{r=iso}) \wedge \\
(\neg \texttt{A} \wedge \neg \texttt{B} \wedge \neg \texttt{C} \longrightarrow \texttt{r=arb})
\end{array}
$$

    where $\texttt{A} \mapsto \texttt{x=y},\ \texttt{B} \mapsto \texttt{y=z},\ \texttt{C} \mapsto \texttt{x=z}$

    (actually: A and B imply C)

Test-Data Generation

- Variants:
  - … Now we can construct a tableau and get by simplification:

case	A	B	C	spec reduces to
(1)	T	T	T	• `r=equ`
(2)	T	T	F	• `r=equ   (!!!)`
(3)	T	F	T	• `r=iso`
(4)	T	F	F	• `r=iso`
(5)	F	T	T	• `r=iso`
(6)	F	T	F	• `r=iso`
(7)	F	F	T	• `r=iso`
(8)	F	F	F	• `r=arb`

Test-Data Generation

- Variants:
  - PHASE III: Borderline analysis.
    Principle: we replace in our DNF inequalities by „the closest values that make the spec true"

$$x \neq y \quad \mapsto \quad x = y + 1 \lor x = y - 1$$

$$x \leq y \quad \mapsto \quad x = y \lor x < y$$

$$x < y \quad \mapsto \quad x = y - 1 \qquad \text{etc.}$$

  - … and recompute the DNF. In general, this gives a much finer mesh …

Test-Data Generation

- Variants:
  - PHASE I: Test for exceptional behaviour.

    We negate the precondition and to DNF generation on the precondition only.

    Test objectives could be:

      - should raise an exception if public
      - should not diverge

Test-Data Generation

- How to handle Recursion ?

Test-Data Generation

Test-Data Generation

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Test-Data Generation

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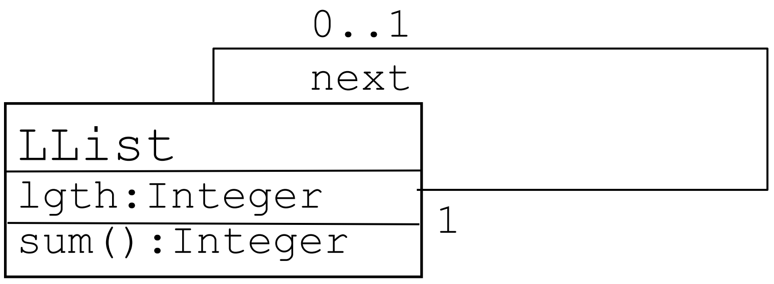

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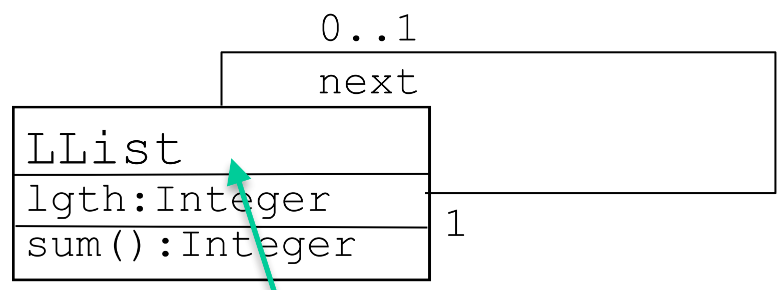

invariant:

$inv_{LList} \equiv \forall$`node∈LList.`

```
node.lgth =if node.next = null
            then 1
            else next.lgth + 1
```

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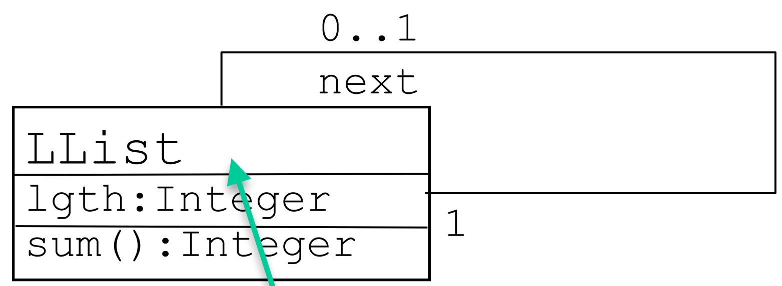

invariant:

$inv_{LList} \equiv \forall \texttt{node} \in \texttt{LList}.$

```
node.lgth =if node.next = null
           then 1
           else next.lgth + 1
```

Note that this excludes cyclic lists !!!

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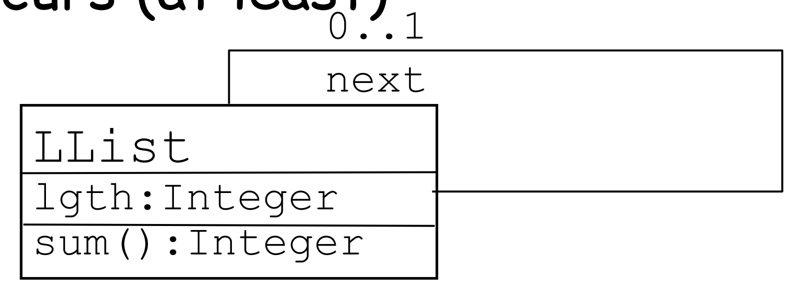

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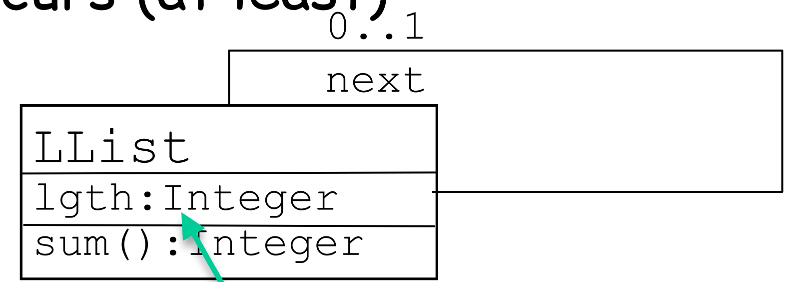

query contract (modifiesOnly({})):

definition $\texttt{pre}_{\texttt{sum}}\texttt{(l)} \equiv \texttt{True}$

definition $\texttt{post}_{\texttt{sum}}\texttt{(l,res)} \equiv$ `res=if l.next=null then l.lgth else l.lgth + l.next.sum()`

definition $\texttt{sum(l)} \equiv \texttt{arb\{r|pre}_{\texttt{sum}}\texttt{(l)} \wedge \texttt{post}_{\texttt{sum}}\texttt{(l,r)\}}$

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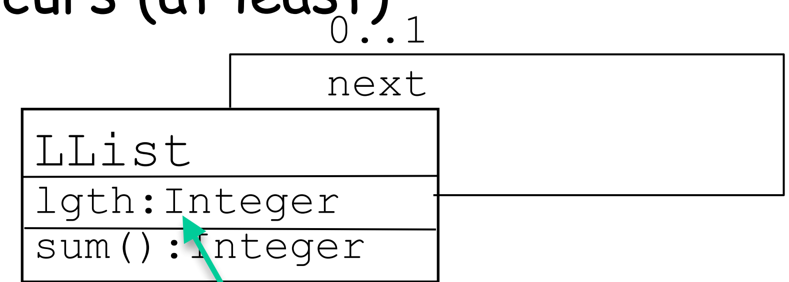

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definition $\texttt{sum(l)} \equiv \texttt{arb\{r|pre}_{\texttt{sum}}\texttt{(l)} \wedge \texttt{post}_{\texttt{sum}}\texttt{(l,r)\}}$

Note that `arb(S)` gives an arbitrary member of `S`: `arb(S)`$\in$`S`. Since from `x=arb({y})` follows `x=y`; thus `sum(l)` is (uniquely) defined.

Test-Data Generation

- Prerequisite: We present the invariant as recursive predicate.

  definition $\mathrm{inv}_{\mathrm{LList\_Core}}$ `n σ ≡(n.lgth(σ) = if n.next(σ)=null then 1`
  `else n.next.lgth(σ) + 1)`

  we have:

  $\mathrm{inv}_{\mathrm{LList}}$ `(σ) = ∀n∈LList(σ).` $\mathrm{inv}_{\mathrm{LList\_Core}}$ `n σ`

  and

  $\mathrm{inv}_{\mathrm{LList\_Core}}$`(n)(σ)= (if n.next(σ)=null then n.lgth(σ) = 1`
  `else n.lgth(σ) =n.next.lgth(σ) + 1`
  `∧ n.next(σ)∈LList(σ)`
  `∧` $\mathrm{inv}_{\mathrm{LList\_Core}}$`(n.next)(σ))`

  Furthermore we have:

  ```
  sum(l)(σ',σ) = if l.next(σ)=null then l.lgth(σ)
                 else l.lgth(σ) + sum(l.next)(σ',σ)
  ```

  We have $\sigma'=\sigma$ (why?). We will again apply (σ',σ) - convention.

Test-Data Generation

- Consider the test specification:

$$X.sum() \equiv Y \qquad \text{(for some } X \in LList\text{, i.e. } X \neq null)$$

$$\equiv inv_{LList}(X) \wedge pre_{sum}(X) \wedge post_{sum}(X,Y)$$

where:

$$pre_{sum}(X) \equiv true$$

$$\begin{aligned} post_{sum}(X,Y) \equiv\ & (\mathtt{if}\ X.next = null\ \mathtt{then}\ Y = X.lgth \\ & \quad \mathtt{else}\ Y = X.lgth + sum(X.next)) \\ \equiv\ & (X.next = null \wedge Y = X.lgth) \\ & \vee (X.next \neq null \wedge Y = X.lgth + sum(X.next)) \end{aligned}$$

Test-Data Generation

Test-Data Generation

- DNF computation yields already the test cases:

$X.sum() \equiv Y$ (for some $X \in LList$, i.e. $X \neq null$)

$\Longrightarrow inv_{LList\_Core}(X) \wedge post_{sum}(X,Y))$

$\equiv$ (if X.next=null then X.lgth = 1

else X.lgth =X.next.lgth+1 ∧ X.next∈LList ∧ inv_{LList_Core}(X.next)) ∧

(if X.next = null then Y = X.lgth

else Y = X.lgth + sum(X.next))

$\equiv$ (if c then C else D elim, DNF)

(X.next=null ∧ X.lgth=1 ∧ Y = X.lgth)

∨ (X.next≠null ∧ X.lgth =X.next.lgth+1

∧ X.next∈LList ∧ inv_{LList_Core}(X.next)

∧ Y = X.lgth+sum(X.next))

Test-Data Generation

- DNF computation yields already the test cases:

X.sum() ≡ Y (for some X∈LList, i.e. X≠null)

$\Longrightarrow$ $\texttt{inv}_{\texttt{LList\_Core}}\texttt{(X)} \wedge \texttt{post}_{\texttt{sum}}\texttt{(X,Y))}$

```
≡ (if X.next=null then X.lgth = 1
   else X.lgth =X.next.lgth+1 ∧ X.next∈LList ∧ inv_LList_Core(X.next)) ∧
  (if X.next = null then Y = X.lgth
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```

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     ∧ X.next∈LList ∧ inv_LList_Core(X.next)
     ∧ Y = X.lgth+sum(X.next))
```

New Test-Case!!

Test-Data Generation

Test-Data Generation

- Intermediate Summary: test-cases known so far ?

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X	Y
	1
...	...
...	...

Test-Data Generation

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X	Y
→ i:LList lgth=1 → null	1
...	...
...	...

Test-Data Generation

- Prerequisite: We present the invariant as recursive predicate.

inv_{LList_Core}`(n)= (if n.next=null then n.lgth = 1`

```
                  else n.lgth =n.next.lgth + 1
                       ∧ n.next∈LList ∧ inv_LList_Core(n.next))
```

- ```
  sum(l) =  if l.next=null then l.lgth
            else l.lgth + sum(l.next)
  ```

```
sum(l) =  if X.next.next=null then X.next.lgth
          else X.next.lgth + sum(X.next.next)
```
```

Test-Data Generation

- DNF computation yields already the test cases:

$X.sum() \equiv Y$ (for some $X \in LList$, i.e. $X \neq null$)

$\Longrightarrow$ ... $\equiv$ ...

$\equiv$ (unfolding sum and inv_{LList_Core})

```
(X.next=null ∧ X.lgth=1 ∧ Y = X.lgth)
  ∨ (X.next≠null ∧ X.lgth=X.next.lgth+1 ∧ X.next∈LList
    ∧ (if X.next.next=null then X.next.lgth = 1
            else X.next.lgth =X.next.next.lgth + 1
                 ∧ X.next.next∈LList ∧ inv_LList_Core(X.next.next))
    ∧ (Y = X.lgth+(if X.next.next=null then X.next.lgth
                      else X.next.lgth + sum(X.next.next)))
```

Test-Data Generation

- DNF computation yields already the test cases:

$X.sum() \equiv Y$ (for some $X \in LList$, i.e. $X \neq null$)

$\Longrightarrow$ ... $\equiv$ ...

$\equiv$ (DNF partial)

```
(X.next=null ∧ X.lgth=1 ∧ Y = X.lgth)
  ∨ (X.next≠null ∧ X.lgth=X.next.lgth+1 ∧ X.next∈LList
    ∧ ( (X.next.next=null ∧ X.next.lgth = 1 ∧ Y = X.lgth+X.next.lgth)
      ∨ (X.next.next≠null ∧ X.next.lgth=X.next.next.lgth+1
        ∧ X.next.next∈LList ∧ inv_LList_Core(X.next.next)
        ∧ Y = X.lgth+ X.next.lgth + sum(X.next.next))
      )
```

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...	...

Test-Data Generation

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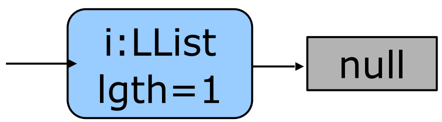
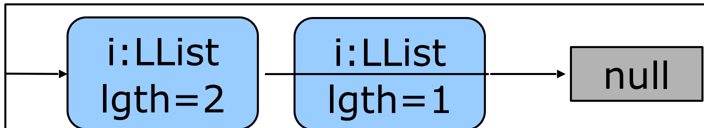
	1
	2
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Summary: Symbolic Test-Case Generation

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    until we are satisfied, i.e. have „enough" test cases ...
  - Select test-data: constraint resolution of test cases.

Test-Data Generation

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$$(\forall X.\ |X|<k \Rightarrow X.sum() \equiv Y)$$
$$\Rightarrow \quad (\forall X.\ X.sum() \equiv Y)$$

where we choose as "complexity mesure" |X|
just X.lgth and k (the number of unfoldings)
was 2 ...

Test-Data Generation

- Coverage Criterion for recursive specification:

  DNF_k

  For all data up to complexity k, we constructed abstract

  test-cases and generated a test.

  In our example, the "complexity measure" is just the length

  of the LLists.

Test-Data Generation

- What are the alternatives to symbolic test-case generation ?

  Must this really be so complicated ???

  Well, think about the probability to “guess” input with a complex invariant or precondition, if you use “blind” random-generation of input...

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  - The process can be tool-supported (HOL-TestGen)
  - The process is intended for automation.

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Test-Data Generation

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  Key-Ingredients are:

  - Unfolding predicates up to a given depth k

  - computing the Disjunctive Normal Form (DNF_k)

  - Adequacy:
    Pick for each test-case (a conjoint in the DNF_k)
    one test, i.e. one substitution for the free variables satisfying the test-case !