

*L3 Mention Informatique  
Parcours Informatique et MIAGE*

# Génie Logiciel Avancé - Advanced Software Engineering

## Annotating UML with MOAL

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# Plan of the Chapter

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- Syntax & Semantics of our own language

## MOAL

- mathematical
- object-oriented
- UML-annotation
- language

(conceived as the „essence“ of annotation languages like OCL, JML, Spec#, ACSL, ...)

# Plan of the Chapter

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- ❑ Concepts of MOAL
  - Basis: Logic and Set-theory
  - MOAL is a Typed Language
  - Basic Types, Sets, Pairs and Lists
  - Object Types from UML
  - Navigation along UML attributes and associations  
(Idea from OCL and JML)
- ❑ Purpose :
  - Class Invariants
  - Method Contracts with Pre- and Post-Conditions
  - Annotated Sequence Diagrams for Scenarios, . . .

# Plan of the Chapter

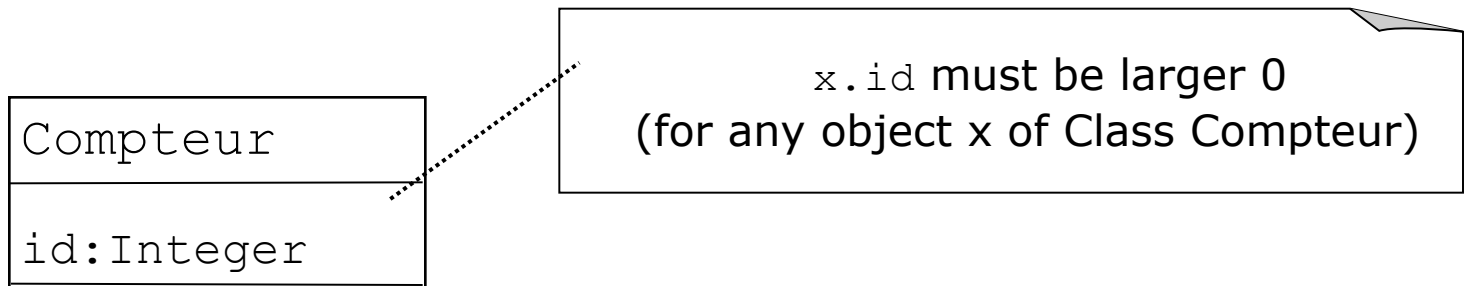
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- ❑ Ultimate Goal:  
Specify system components to improve analysis, design, test and verification activities
- ❑ . . . understanding how some analysis tools work . . .
- ❑ . . . understanding key concepts such as class invariants and contracts for analysis and design

# Motivation: Why Logical Annotations

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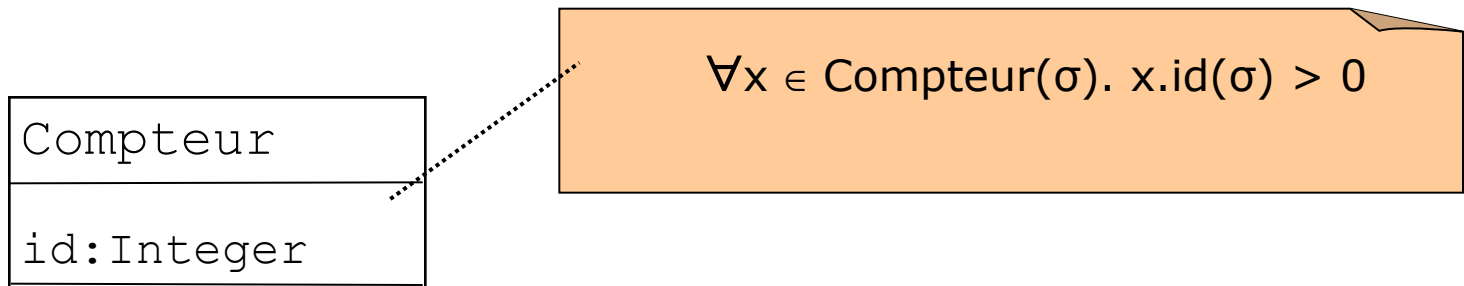
- More precision needed (like JML, VCC) that constrains an underlying **state  $\sigma$**



# Motivation: Why Logical Annotations

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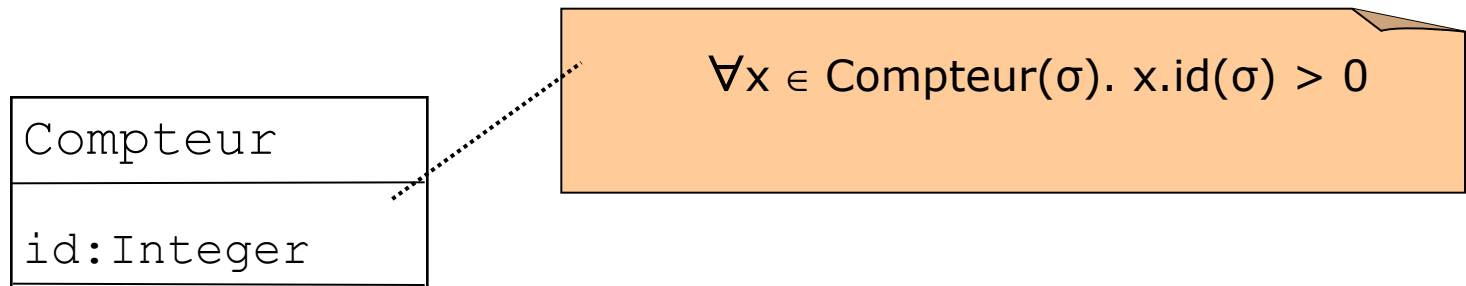
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# Motivation: Why Logical Annotations

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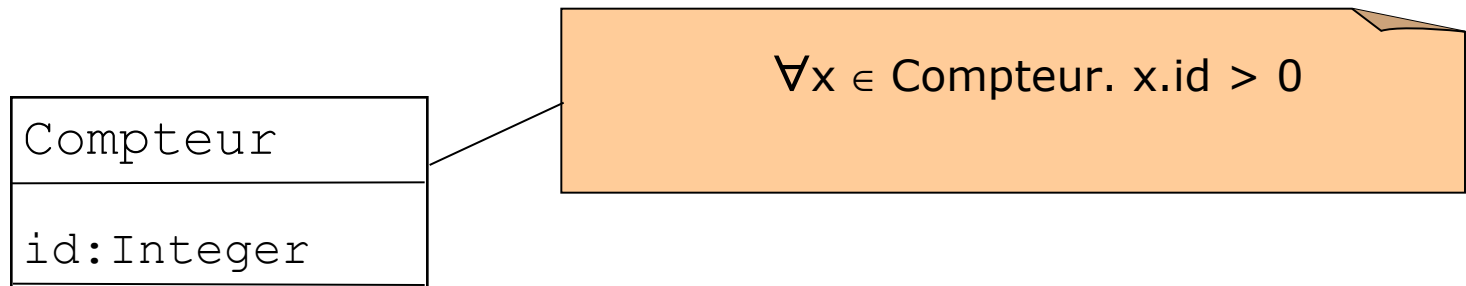
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# Motivation: Why Logical Annotations

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- More precision needed  
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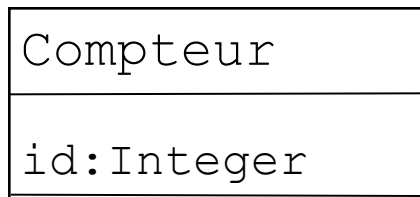
... by abbreviation convention if no confusion arises.



# Motivation: Why Logical Annotations

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- More precision needed  
(like JML, VCC) that constrains an underlying **state  $\sigma$**



definition  $\text{inv}_{\text{Compteur}}(\sigma) \equiv \forall x \in \text{Compteur}(\sigma). x.\text{id}(\sigma) > 0$

... or by convention

definition  $\text{inv}_{\text{Compteur}} \equiv \forall x \in \text{Compteur}. x.\text{id} > 0$

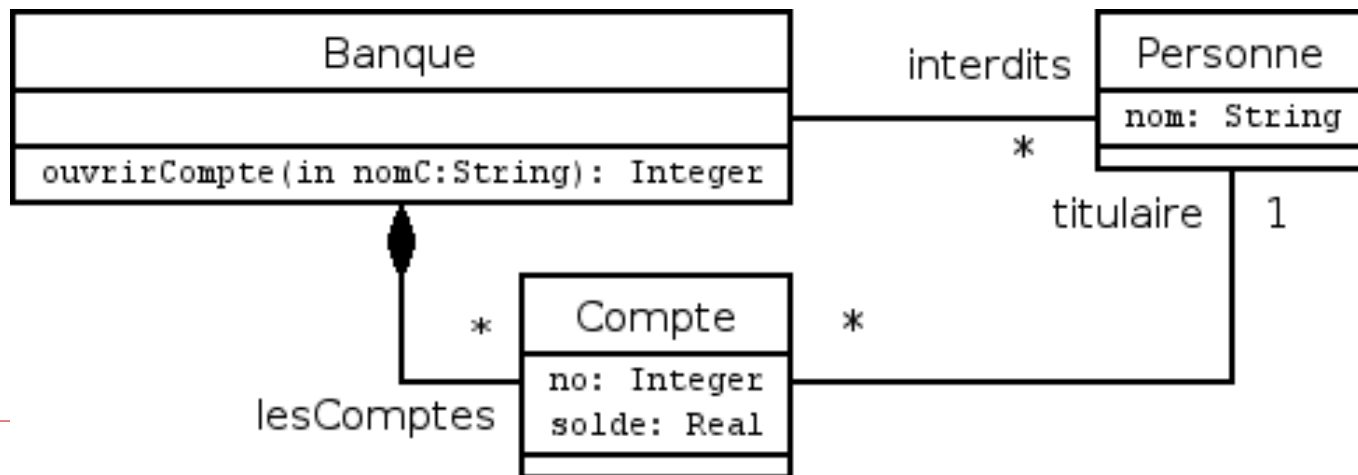
... or as mathematical definition in a separate document or text ...

# A first Glance to an Example: Bank

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Opening a bank account. Constraints:

- ❑ there is a blacklist
- ❑ no more overdraft than 200 EUR
- ❑ there is a present of 15 euros in the initial account
- ❑ account numbers must be distinct.



# A first Glance to an Example: Bank (2)

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**definition** `unique`  $\equiv$  `isUnique(.no) (Compte)`

**definition** `noOverdraft`  $\equiv \forall c \in \text{Compte}. c.\text{id} \geq -200$

**definition** `preouvrirCompte` (`b:Banque, nomC:String`)  $\equiv$   
 $\forall p \in \text{Personne}. p.\text{nom} \neq \text{nomC}$

**definition** `postouvrirCompte` (`b:Banque, nomC:String, r::Integer`)  $\equiv$   
 $|\{p \in \text{Personne} \mid p.\text{nom} = \text{nomC} \wedge p.\text{isNew}()\}| = 1$   
 $\wedge |\{c \in \text{Compte} \mid c.\text{titulaire}.\text{nom} = \text{nomC}\}| = 1$   
 $\wedge \forall c \in \text{Compte}. c.\text{titulaire}.\text{nom} = \text{nomC} \rightarrow c.\text{solde} = 15$   
 $\wedge \text{isNew}(c)$

# MOAL: a specification language?

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- In the following, we will discuss the

MOAL Language in more detail ...

# Syntax and Semantics of MOAL

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## □ The usual logical language:

- True, False
- negation :  $\neg E$ ,
- or:  $E \vee E'$ , and:  $E \wedge E'$ , implies:  $E \rightarrow E'$
- $E = E'$ ,  $E \neq E'$ ,
- if  $C$  then  $E$  else  $E'$  endif
- let  $x = E$  in  $E'$
  
- Quantifiers on sets and lists:

$\forall x \in \text{Set}. P(x)$

$\exists x \in \text{Set}. P(x)$

# Syntax and Semantics of MOAL

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- MOAL is (like OCL or JML) a typed language.
  - Basic Types:  
Boolean, Integer, Real, String
  - Pairs:  $X \times Y$
  - Lists: List(X)
  - Sets: Set(X)

# Syntax and Semantics of MOAL

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- The arithmetic core language.  
expressions of type Integer or Real:
  - $1, 2, 3 \dots$  resp.  $1.0, 2.3, \pi$ .
  - $- E, E + E',$
  - $E * E', E / E',$
  - $\text{abs}(E), E \text{ div } E', E \text{ mod } E' \dots$

# Syntax and Semantics of MOAL

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- The expressions of type `String`:
  - `S concat S'`
  - `size(S)`
  - `substring(i, j, S)`
  - `'Hello'`



# Syntax and Semantics of MOAL Sets

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- $| S |$  size as Integer
- $\text{isUnique}(f)(S) \equiv \forall x, y \in S. f(x)=f(y) \rightarrow x=y$
- $\{\}, \{a, b, c\}$  empty and finite sets
- $e \in S, e \notin S$  is element, not element
- $S \subseteq S'$  is subset
- $\{x \in S \mid P(x)\}$  filter
- $S \cup S', S \cap S'$  union, intersect  
between sets of same type
  
- Integer, Real, String ...  
are symbols for the set  
of all Integers, Reals, ...

# Syntax and Semantics of MOAL Pairs

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- $(X, Y)$  pairing
- $\text{fst}(X, Y) = X$  projection
- $\text{snd}(X, Y) = Y$  projection

# Syntax and Semantics of MOAL Lists

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Lists  $S$  have the following operations:

- $x \in L$  -- is element (overload!)
- $|S|$  -- length as Integer
- $\text{head}(L), \text{last}(L)$
- $\text{nth}(L, i)$  -- for  $i$  between 0 et  $|S| - 1$
- $L@L'$  -- concatenate
- $e\#S$  -- append at the beginning
- $\forall x \in \text{List}. P(x)$  -- quantifiers :
- $[x \in L \mid P(x)]$  -- filter
- Finally, denotations of lists:  $[1,2,3], \dots$

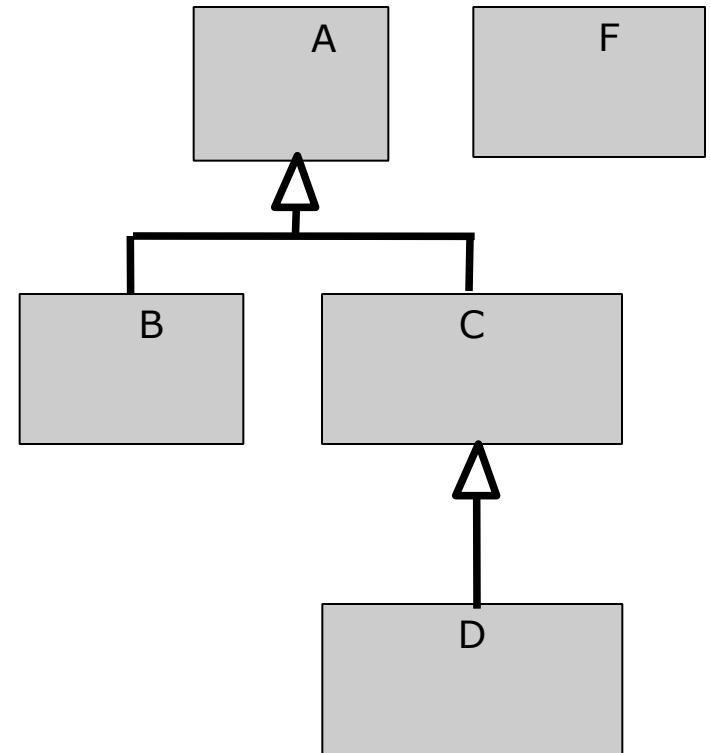
# Syntax and Semantics of Objects

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- ❑ Objects and Classes follow the semantics of UML
  - inheritance / subtyping
  - casting
  - objects have an id
  - NULL is a possible value in each class-type
  - for any class A, we assume a function:

$$A(\sigma)$$

which returns the set of objects of class A in state  $\sigma$  (the « instances » in  $\sigma$ ).

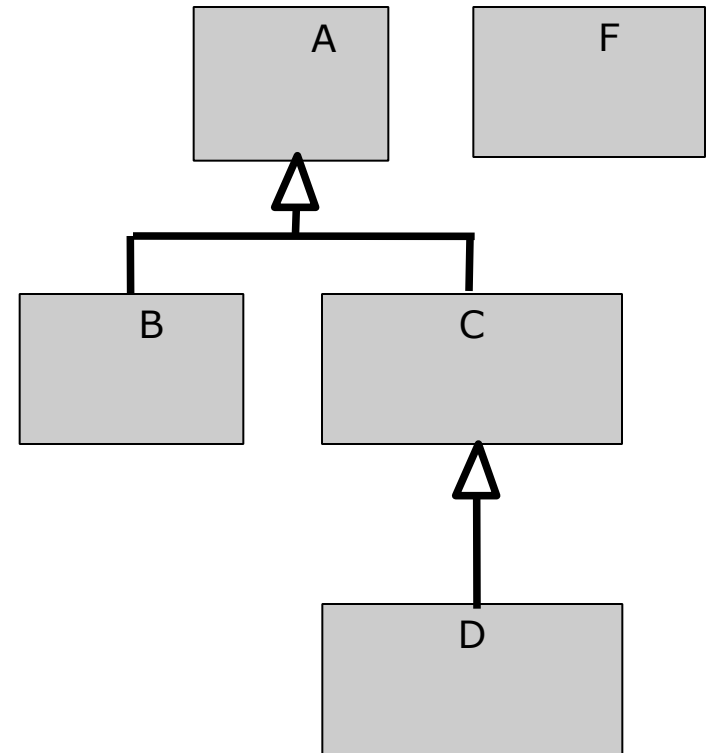


# Syntax and Semantics of Objects

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- ❑ Objects and Classes follow the semantics of UML

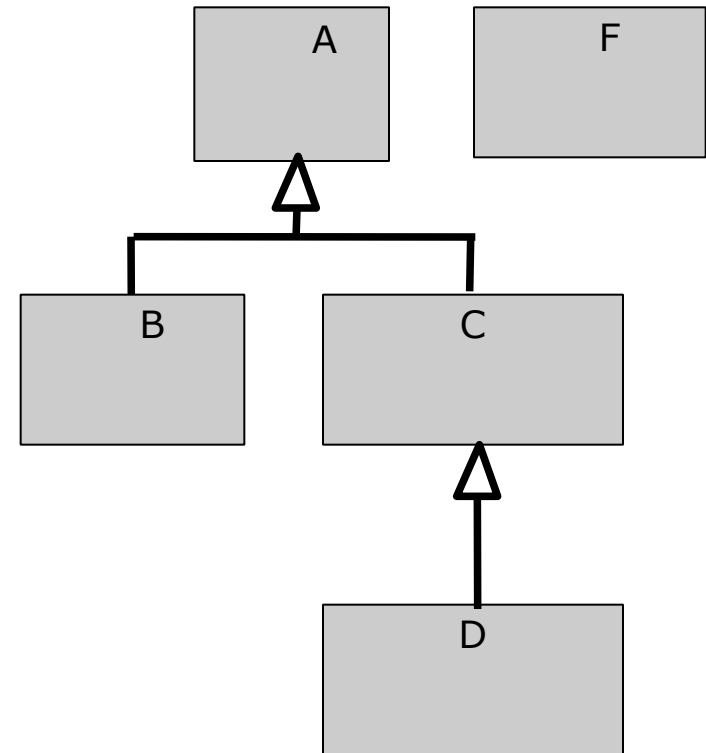
Recall that we will drop the index ( $\sigma$ ) whenever it is clear from the context



# Syntax and Semantics of Objects

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- ❑ As in all typed object-oriented languages casting allows for converting objects.
- ❑ Objects have two types:
  - the « apparent type »  
(also called static type)
  - the « actual type »  
(the type in which an object was created)
  - casting changes the apparent type along the class hierarchy, but not the actual type



# Syntax and Semantics of Objects

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➤ Assume the creation of objects

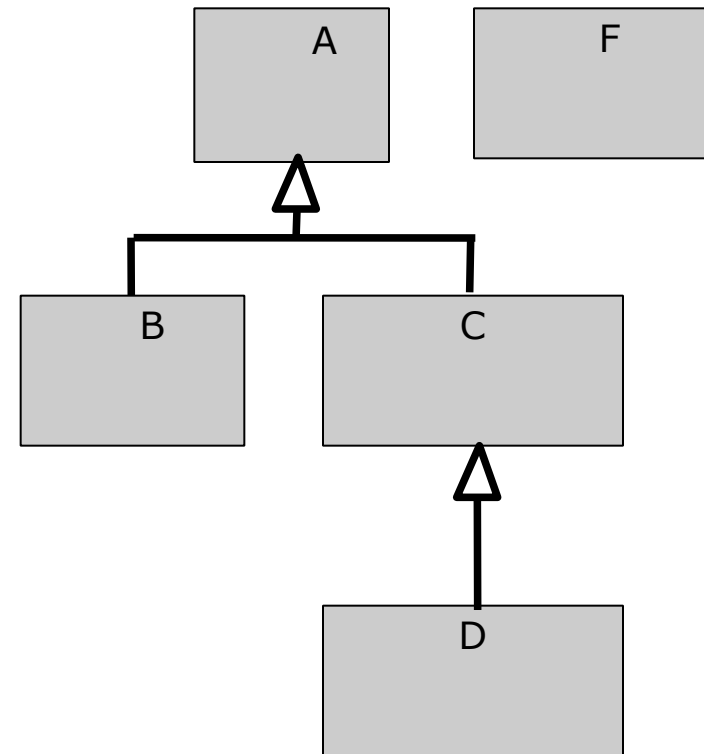
a in class A, b in class B,  
c in class C, d in class D,

➤ Then casting:

`<F>b` is illtyped

`<A>b` has apparent type A,  
but actual type B

`<A>d` has apparent type A,  
but actual type D



# Syntax and Semantics of OCL / UML

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- We will also apply cast-operators to an entire set: So

$\langle A \rangle B(\sigma)$  (or just:  $\langle A \rangle B$ )  
is the set of instances  
of  $B$  casted to  $A$ .

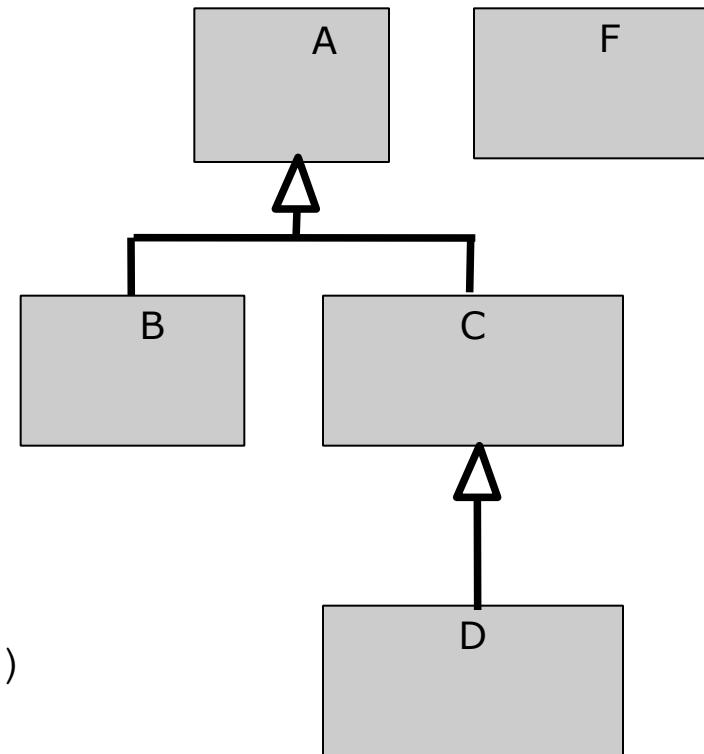
We have:

$$\langle A \rangle B \cup \langle A \rangle C \subseteq A$$

but:

$$\langle A \rangle B \cap \langle A \rangle C = \{\}$$

and also:  $\langle A \rangle D \subseteq A$  (for all  $\sigma$ )





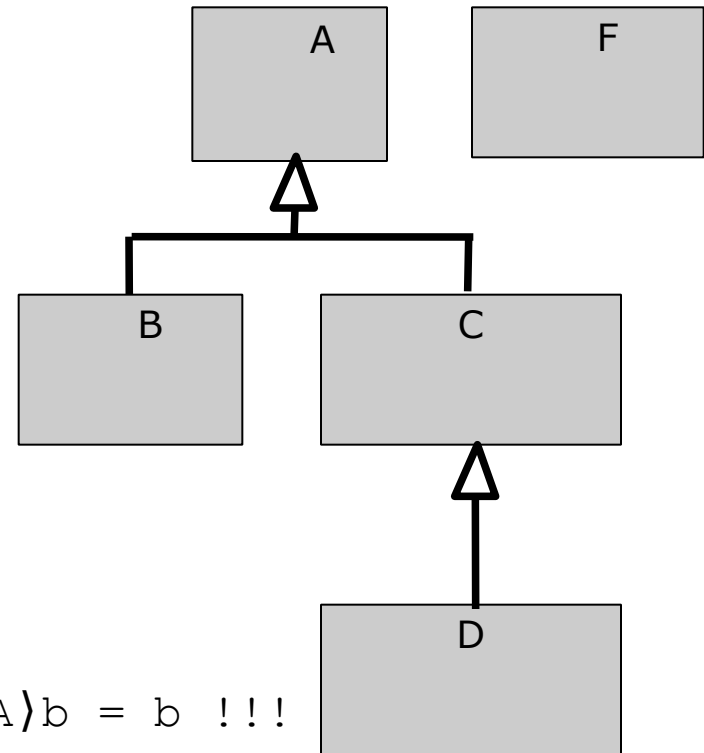
# Syntax and Semantics of Objects

- Instance sets can be used to determine the actual type of an object:

$x \in B$

corresponds to Java's `instanceof` or OCL's `isKindOf`. Note that casting does NOT change the actual type:

$\langle A \rangle b \in B$ , and  $\langle B \rangle \langle A \rangle b = b$  !!!



# Syntax and Semantics of Objects

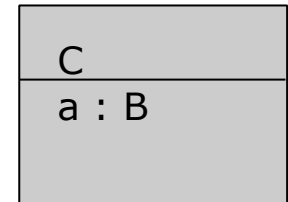
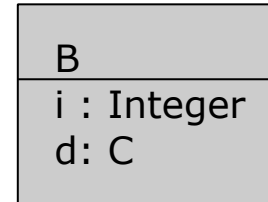
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- Summary:
  - there is the concept of **actual** and **apparent** type  
(anywhere outside of Java: **dynamic** and **static** type)
  - type tests check the former
  - type casts influence the latter,  
but not the former
  - up-casts possible
  - down-casts invalid
  - consequence:  
up-down casts are identities.

# Syntax and Semantics of Object Attributes

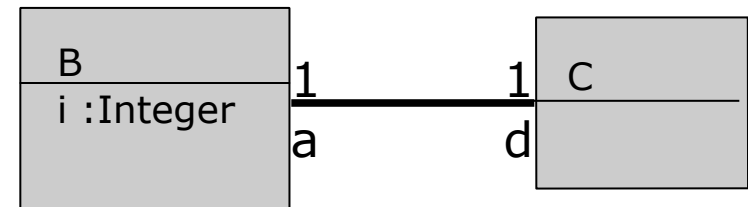
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- Objects represent structured, typed memory in a state  $\sigma$ . They have **attributes**.



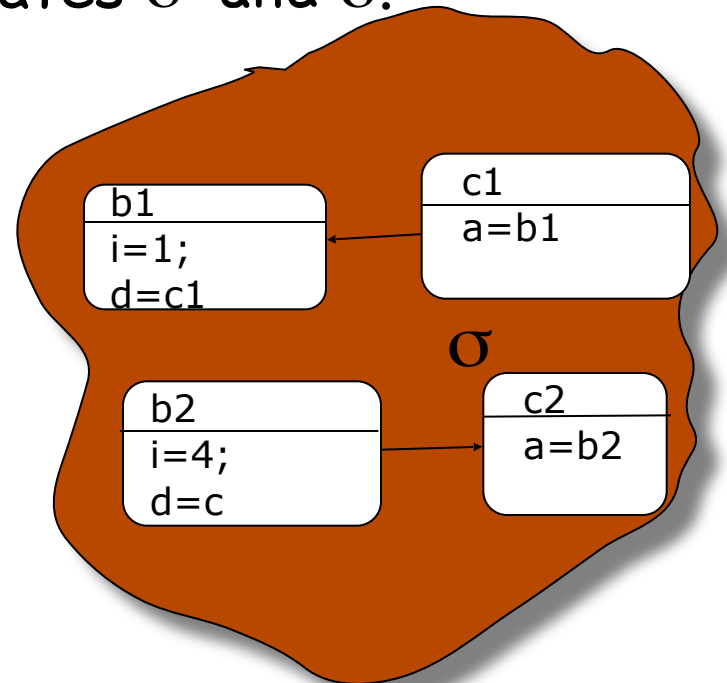
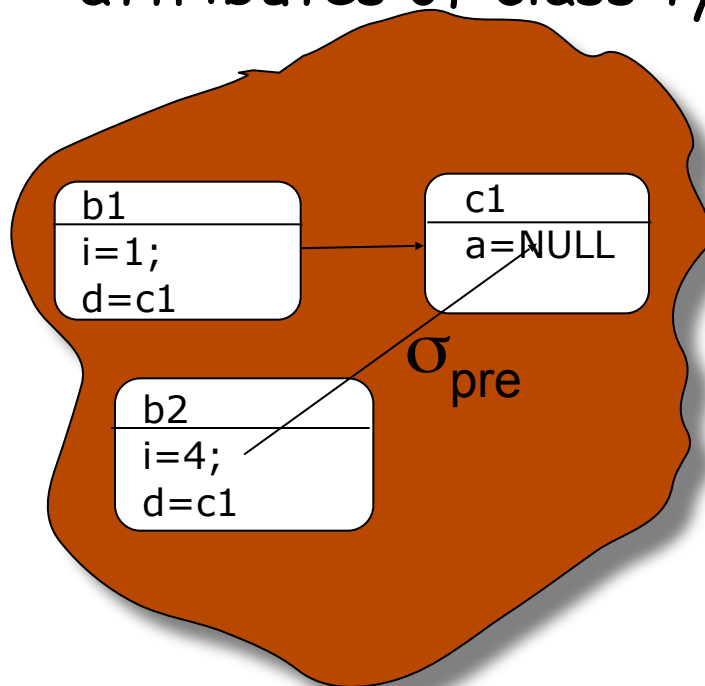
They can have class types.

- Reminder: In class diagrams, this situation is represented traditionally by Aggregations (somewhat sloppily: Associations)



# Syntax and Semantics of Object Attributes

- Example:  
attributes of class type in states  $\sigma'$  and  $\sigma$ .



# Syntax and Semantics of Object Attributes

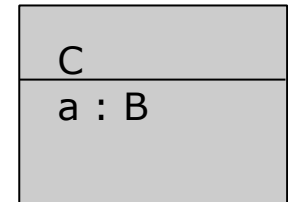
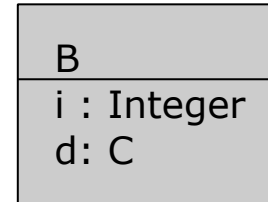
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- each attribute is represented by an *accessor-function* in MOAL. The class diagram right corresponds to the declaration of them:

$.i(\sigma) :: B \rightarrow \text{Integer}$

$.a(\sigma) :: C \rightarrow B$

$.d(\sigma) :: B \rightarrow C$



- This makes navigation expressions possible:

➤  $b1.d(\sigma) :: C$

$c1.a(\sigma) :: B$

$b1.d(\sigma).a(\sigma).d(\sigma).a(\sigma) \dots$

# Syntax and Semantics of Object Attributes

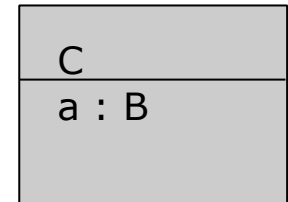
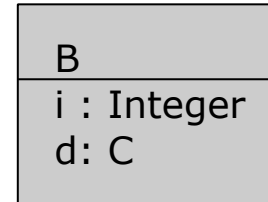
---

- each attribute is represented by a function in *MOAL*.  
The class diagram right corresponds to declaration of accessor functions:

`.i( $\sigma$ ) :: B -> Integer`

`.a( $\sigma$ ) :: C -> B`

`.d( $\sigma$ ) :: B -> C`



- Applying the  $\sigma$ -convention, this makes the following navigation expression syntax possible:

➤ `b1.d :: C`

`c1.a :: B`

`b1.d.a.d.a ...`

# Syntax and Semantics of Object Attributes

---

- ❑ Assessor functions “dereferentiate” pointers in a given state
- ❑ Accessor functions of class type are **strict** wrt. NULL.

➤  $\text{NULL}.d = \text{NULL}$   
 $\text{NULL}.a = \text{NULL}$

- Note that navigation expressions depend on their underlying state:

$b1.d(\sigma_{\text{pre}}).a(\sigma_{\text{pre}}).d(\sigma_{\text{pre}}).a(\sigma_{\text{pre}}) = \text{NULL}$

$b1.d(\sigma).a(\sigma).d(\sigma).a(\sigma) = b1$  !!!

(cf. Object Diagram pp 28)

# Syntax and Semantics of Object Attributes

---

- ❑ Assessor functions “dereferentiate” pointers in a given state
- ❑ Accessor functions of class type are **strict** wrt. NULL.
  - > `NULL.d = NULL`  
`NULL.a = NULL`
  - > The  $\sigma$  convention allows to write :

`old(b1.d.a.d.a) = NULL`  
`b1.d.a.d.a = b1` !!!

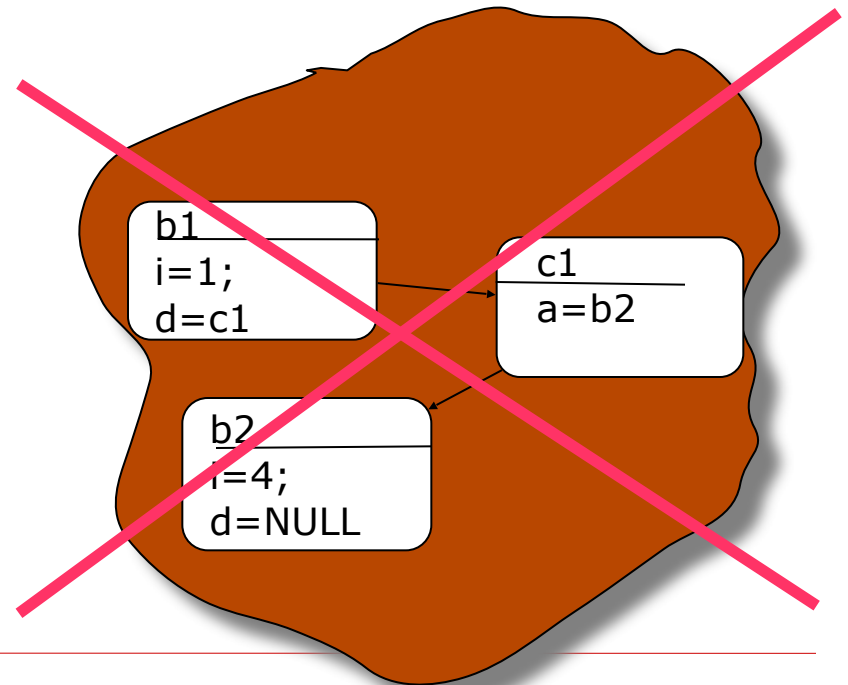
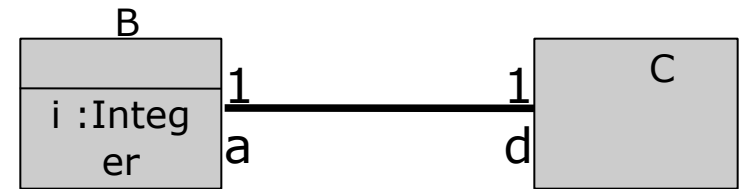
(cf. Object Diagram pp 28)



# Syntax and Semantics of Object Attributes

- Note that associations are meant to be « relations » in the mathematical sense. (Here, we treat them like aggregations, which is strictly speaking a design step)

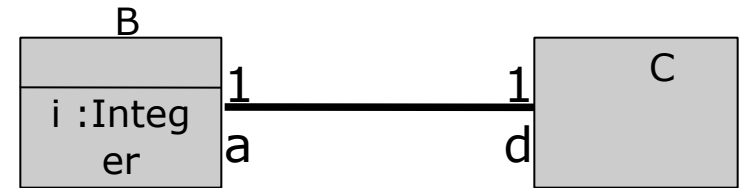
Thus, states (object-graphs) of this form do not represent an association of the cardinality 1 - 1:



# Syntax and Semantics of Object Attributes

- This is reflected by 2  
« association integrity  
constraints ».

For the 1-1-case, they are:



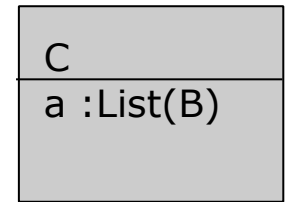
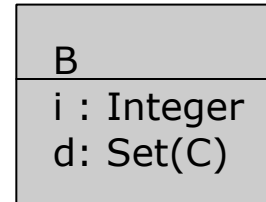
➤ definition  $ass_{B.d.a} \equiv \forall x \in B. x.d.a = x$

➤ definition  $ass_{C.a.d} \equiv \forall x \in C. x.a.d = x$

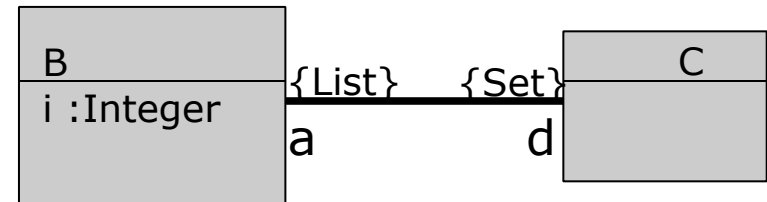
# Syntax and Semantics of Object Attributes

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- Attributes can be Lists or Sets of class types:



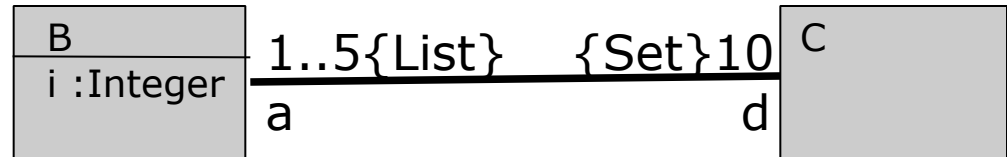
- Reminder: In class diagrams, this situation is represented traditionally by Associations (equivalent)



- In analysis-level Class Diagrams, the type information is still omitted; due to overloading of  $\forall x \in X. P(x)$  etc. this will not hamper us to specify ...

# Syntax and Semantics of Object Attributes

- Cardinalities in Associations can be translated canonically into MOCL invariants:



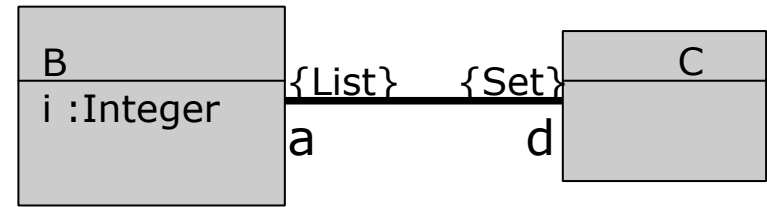
➤ definition  $\text{card}_{B.d} \equiv \forall x \in B. |x.d| = 10$

➤ definition  $\text{card}_{C.a} \equiv \forall x \in C. 1 \leq |x.a| \leq 5$

# Syntax and Semantics of Object Attributes

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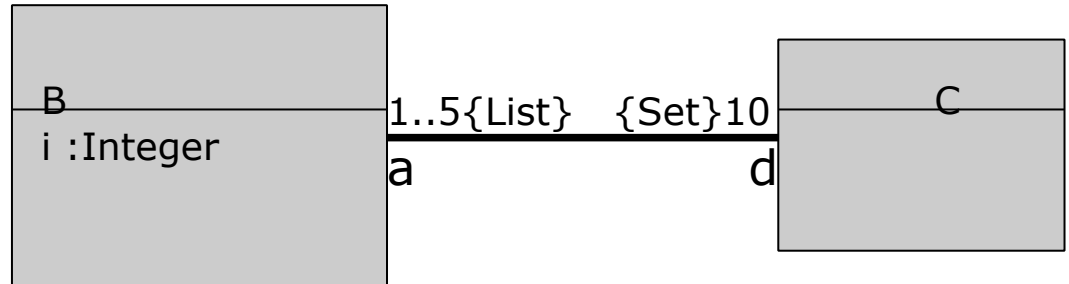
- ❑ Accessor functions are defined as follows for the case of NULL:



- $\text{NULL}.d = \{\}$  -- mapping to the neutral element
- $\text{NULL}.a = []$  -- mapping to the neutral element.

# Syntax and Semantics of Object Attributes

- Cardinalities in Associations can be translated canonically into MOCL invariants:

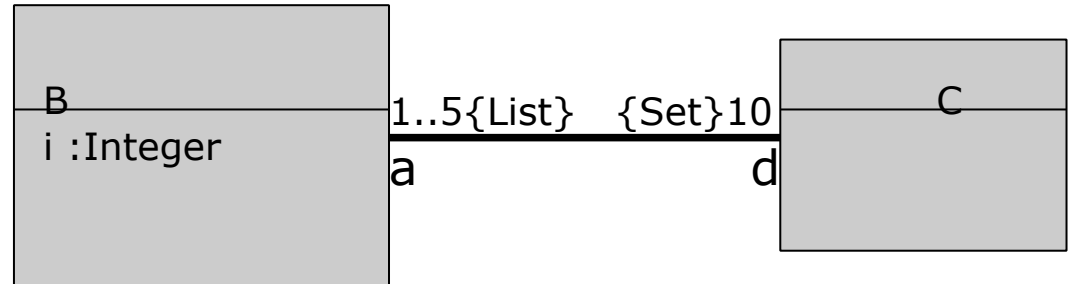


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# Syntax and Semantics of Object Attributes

- The corresponding association integrity constraints for the \*-\*-case are:



➤ definition  $ass_{B.d.a} \equiv \forall x \in B. x \in x.d.a$

➤ definition  $ass_{C.a.d} \equiv \forall x \in C. x \in x.a.d$

# Summary

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- ❑ MOAL makes the UML to a real, formal specification language
- ❑ MOAL can be used to annotate Class Models, Sequence Diagrams and State Machines
- ❑ Working out, making explicit the constraints of these Diagrams is an important technique in the transition from Analysis documents to Designs.