POLYTECH PARIS-SUD

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Cycle Ingénieur – 2ème année

Département Informatique

Verification and Validation

Part IV :

Deductive Verification (I)

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Recall: Validation and Verification

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- Limits in perfection:

  We know only up to a given “certainty” that the
  program meets the specification ...

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  - the tools validating them use internally automated proofs

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Nous, on le fera à la main ...

How to do Verification ?

In the sequel, we concentrate on

Deductive Verification

(Proof Techniques)

Standard example

The specification in UML/MOAL (Classes in USE Notation):

```
class Triangles inherits_from Shapes
   attributes
      a : Integer
      b : Integer
      c : Integer


   operations
      mk(Integer,Integer,Integer):Triangle
      is_Triangle(): triangle
   end
```

Standard example : Triangle

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```
context Triangles:
inv def      : a.oclIsValid() and b.oclIsValid()...
inv pos      : 0<a and 0<b and 0<c
inv triangle : a+b>c  and  b+c>a  and  c+a>b

context Triangle::isTriangle()
post equi : a=b and b=c implies result=equilateral
post iso  : ((a<>b or b<>c) and
             (a=b or b=c or a=c))implies result=isosceles
post default: (a<>b or b<>c) and
               (a<>b and b<>c and a<>c)
               implies result=arbitrary
```

Standard example: Triangle

```
procedure triangle(j,k,l : positive) is
 eg: natural := 0;
begin
if  j + k <= l or k + l  <= j or l + j <= k then
    put("impossible");
else if  j = k  then   eg := eg + 1;  end if;
     if  j = l  then   eg := eg + 1;  end if;
     if  l = k  then   eg := eg + 1;  end if;
      if  eg = 0  then  put("quelconque");
     elsif   eg = 1  then put("isocele");
     else    put("equilateral");
     end if;
end if;
end triangle;
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These programs have the following characteristics:

- one is more efficient, but more complex
- But both have the same specification !

How to do Verification ?

How to PROVE that programs

meet the specification ?

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  A rule with n=0 is an *elementary fact*. Variables occurring in the formulas A_n can be arbitrarily substituted.
- Assumptions and conclusions are terms in a logic containing variables

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- where the first rule "reflexivity" is an elementary fact.

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$$\frac{1 + 2 = 2 + 1 \quad 2 + 1 = 3}{1 + 2 = 3}$$

{x↦1+2,
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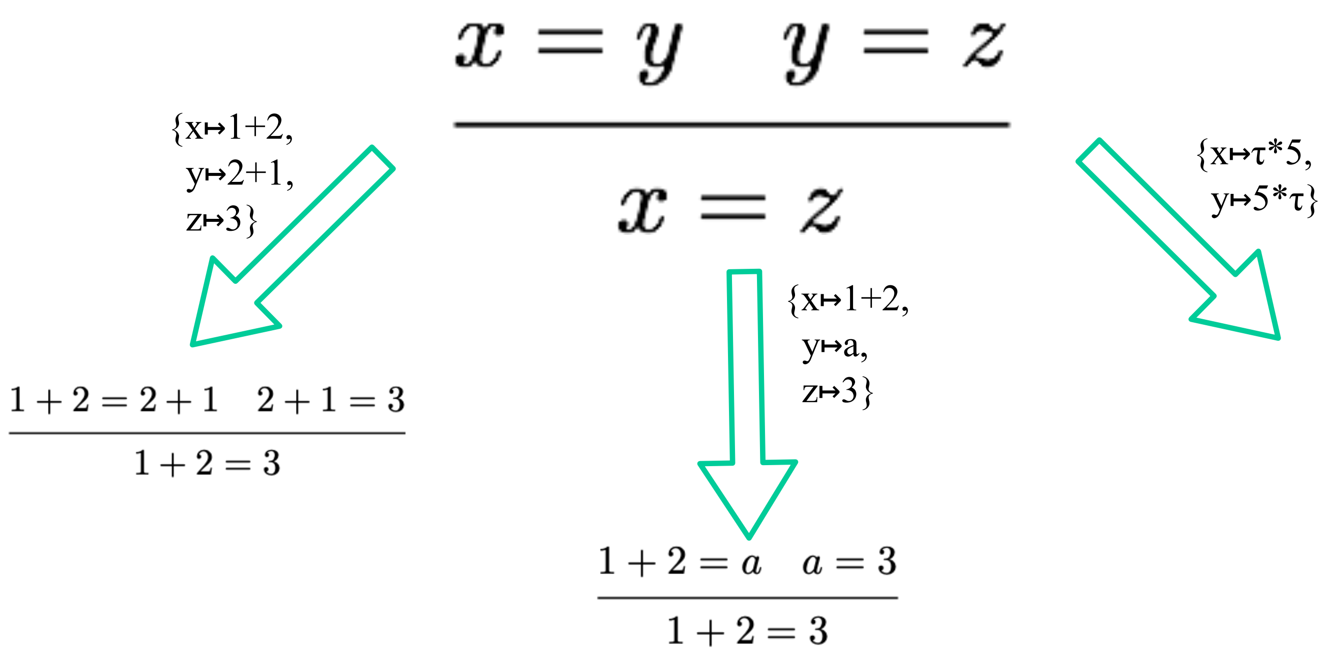

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{x↦τ*5, y↦5*τ}

$$\frac{\tau * 5 = 5 * \tau \quad 5 * \tau = z}{\tau * 5 = z}$$

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$$\dfrac{\dfrac{f(a,b)=a}{a=f(a,b)} \qquad \dfrac{f(a,b)=a \quad f(f(a,b),b)=c}{f(a,b)=c}}{a=c} \qquad \dfrac{}{g(a)=g(a)}$$

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- The non-elementary facts at the leaves are the *global assumptions* (here $f(a,b) = a$ and $f(f(a,b),b) = c$).

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- This is what theorems are: derivable facts from assumptions in a certain logical system ...

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$$\frac{A}{A \vee B} \quad \frac{B}{A \vee B}$$

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- PL + E + Arithmetics (A) in so-called natural deduction:

$$\frac{\neg\neg A}{A} \qquad \frac{A}{\neg\neg A} \qquad \frac{A \quad B}{A \wedge B} \qquad \frac{A \wedge B \qquad \begin{matrix}[A, B] \\ \vdots \\ Q\end{matrix}}{Q} \qquad \frac{False}{A}$$

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Hoare – Logic: A Proof System for Programs

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Logic for Programs ???

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- We consider the Hoare-Logic (Sir Anthony Hoare ...), technically an inference system PL + E + A + Hoare

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  - the sequential compos. c_1 ; c_2
  - the conditional IF cond THEN c_1 ELSE c_2
  - the loop WHILE cond DO c

  where c, c_1, c_2, are cmd's, V variables,

  E an arithmetic expression, and cond a boolean expression.

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  - *P* and *Q* are formulas over the variables *V*, so they can be seen as set of possible states.

Hoare Logic vs. Symbolic Execution

- Hoare Logic is also based notion of a *symbolic state.*

$$state_{sym} = V \rightarrow Set(D)$$

Hoare Logic vs. Symbolic Execution

- Intuitively:

$$\vdash \{P\}\ cmd\ \{Q\}$$

means:

If a program cmd starts in a state admitted by P if it terminates, that the program must reach a state that satisfies Q.

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- Makes Test-Hypothesis, which can be hard to justify !

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- *Makes assumptions on language, method, tool-correctness, too !*

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  But at least, we can gain confidence validating specs, i.e. by animation and test, thus, by experimenting with them ...

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